ON GRAIN-SIZE-DEPENDENT CHEMICAL FRACTIONATION
IN THE EARLY PROTO-SOLAR CLOUD

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Abstract. The fractionation problem of minerals in a turbulent protosolar disk is extended to a non-constant grain size due to evaporation and recondensation. Numerical and approximate analytical solutions are given.

1. Introduction

A new theory for chemical fractionation in a turbulent proto-planetary accretion disk has been developed by Morrill (1983, 1985) and Morrill and Völk (1984; hereafter referred to as MV). In their model grains are transported in turbulent eddies and at the same time drift toward the centre of the disk as a result of both standard viscous evolution of the gas and angular momentum loss associated with gas drag. In this way they migrate into hotter regions. When the sublimation temperature of a certain mineral constituent is reached, the vapour of this mineral will be released at that radial distance. It is an important point in the theory of MV that the vapour can also be transported back outward to cooler regions by the action of turbulent diffusion. In these cooler regions it recondenses onto the grains that are present there.

For the sake of an analytical treatment three simplifications were made by MV which we want to avoid in this paper: (1) their solutions were given for a particular grain size only and (2) the grain size and (3) the time-scale of recondensation were assumed constant with radial distance. However, as a result of recondensation the size of the grains grows, and this produces a nonlinear coupling to the size-dependent drift velocity and recondensation properties appearing in the transport equations. We are interested in mathematical properties of the solutions and here present and discuss numerical results for two test cases, where these effects are considered within the framework of a self-consistent calculation. We also give new analytical results for an arbitrary grain-size.

2. Method

To simplify matters we follow MV who assumed that all grains at distance r from the central proto-Sun have exactly the same mass m(r). This approach ignores the size distribution of grains that would be produced by effects like collisional coagulation and

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disruption, but it enables the system of moment equations of the transport equation to be closed. In a forthcoming contribution we shall improve the theory also in this point. MV derived a system of drift-diffusion equations for the grain number density \( n(r) \) (grains \( \text{cm}^{-3} \)), the solid mass density \( \rho_{ks}(r) \) (g \( \text{cm}^{-3} \)), and the vapour density \( \rho_{kv}(r) \) (g \( \text{cm}^{-3} \)) of each mineral constituent \( k \). The *drift velocity* \( v_s \) of the grains is size-dependent and given by

\[
v_s = - \left( \frac{v}{r} \right) (1 + \lambda),
\]

with \( \lambda \) a dimensionless size parameter which is proportional to the grain radius \( a \), and \( v \) the turbulent viscosity (which also serves as diffusion coefficient throughout) of the disk. Equation (1) is only valid for an accretion disk model when the opacity \( \kappa \) is dominated by small grains \( (\kappa \propto T^2) \) (e.g., Lin, 1981) and the grains are well-coupled to the turbulent velocity field. With the standard values from MV for a proto-Sun with mass \( M_c = \frac{1}{2} M_o \), we have \( \lambda \approx 0.1a \) (see Equation (A13) in MV). The drift velocity for vapour components is given by \( v_g = - \frac{v}{r} \).

In a stationary disk the (height-averaged) temperature increases monotonically towards the centre (for out disk \( T \propto r^{-3/2} \)) due to equilibrium between viscous energy dissipation and radiative losses. Grains drifting towards the centre will cross the sublimation temperatures \( T_k \) of the constituent minerals one after the other. Because the evaporation time-scale at \( T_k \) is so much shorter than the drift and diffusion time-scales, we assume instant evaporation. At the cool edge of a sublimation boundary the vapour transported there by turbulent diffusion recondenses rapidly onto the grains and increases their mass. The recondensation effect is also size-dependent \( (\propto n a^2) \), which produces a non-linear coupling between the transport equations. The recondensation time-scale will be taken consistent with the radial particle distribution in a disk with \( M_c = \frac{1}{2} M_o \) and an accretion rate of \( \dot{M} = 10^{-6} M_o \text{ yr}^{-1} \).

The grain mass is defined by

\[
m(r) = \sum_{k=1}^{N} \frac{\rho_{ks}(r)}{n(r)}
\]

(note that \( a \propto m^{1/3} \)), with \( N \) the number of mineral species in the grain. The minerals are numbered in a way that the most refractory is \( k = 1 \) and the most volatile is \( k = N \). To demonstrate the effect that this paper addresses, we restrict ourselves to two vaporizable minerals and a very refractory tiny residuum, which does not take part in the recondensation cycle, but serves to maintain grains of finite mass up to the total sublimation temperature \( T_1 \). The problem is solved by numerical iterations. We start with an uniform mass in each of the following radius intervals (in decreasing temperature order): zone 1 \((r_1, r_2)\), zone 2 \((r_2, r_3)\), and zone 3 \((r_3, R)\), with \( r_k \) the sublimation radius of mineral \( k (r_k \propto T_k^{2/3}) \). In zone 3 the grain mass \( m_0 \) is fixed by the choice of an initial size parameter \( \lambda_0 \) (stemming from, e.g., consideration of the coagulation process). We define normalized abundances \( A_k = f_k/f_0 \), with \( f_k \) the abundances of mineral \( k \) and \( f_0 \) the interstellar dust-to-gas ration \( (f_0 = 0.02) \). In terms of \( A_k \), the initial mass is \( (1 - A_k)m_0 \) in zone 2 and \( (1 - A_3 - A_2)m_0 \) in zone 1. Starting with this initial radial