THE ROLE OF THE CORIOLIS FORCE
ON THE STABILITY OF ROTATING MAGNETIC STARS
AND THE ORIGIN OF CONVECTIVE MOTIONS

KUNITOMO SAKURAI*

Radio Astronomy Branch, Laboratory for Extraterrestrial Physics,
NASA, Goddard Space Flight Center, Greenbelt, Md. U.S.A.

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Abstract. Based on the method of the energy principle, the effect of the Coriolis force in the stability of rotating magnetic stars is examined and the conditions for instability is derived. It is shown that, in these stars, the effect of this force is to inhibit the onset of convective motion.

Discussion is given on the possibility of hydromagnetic dynamo processes in respect to the convective motion inside these stars.

1. Introduction

At present, it is thought of that the magnetism of the Earth and magnetic stars like the Sun is generated and maintained by the hydromagnetic self-exciting dynamo mechanism in their interiors (e.g., Bullard and Gellman, 1954; Elsasser, 1956; Babcock, 1961; Parker, 1970a, b). Furthermore, this mechanism is thought of as being necessarily related to the rotating motions of such stars and planets. This idea, therefore, assumes that the convective motions in their interiors are generated in association with their rotating motion.

In this paper, the effect of the Coriolis force on the onset of convective motions inside rotating stars will be considered using the energy principle first derived by Bernstein et al. (1958) for the non-rotating systems. Then, discussion will be given on the relation of instability for convective motions with the hydromagnetic dynamo processes.

2. Rotating Magnetic Stars and Their Stability

When a rotating magnetic star is in hydromagnetic equilibrium, this state is expressed by

\[- \nabla P + \frac{1}{c} j \times B + \mathbf{g} \Omega \times (\Omega \times r),\]

(1)

\[\text{curl} \mathbf{B} = \frac{4\pi}{c} \mathbf{j},\]

(2)

and

\[\text{div} \mathbf{B} = 0,\]

(3)

where \( \rho, \Omega, r, P, c, j, B \) and \( \mathbf{g} \) are respectively the mass density, the angular velocity,
the position vector, the pressure, the speed of light, the electric current, the magnetic field and the gravity force.

If we take into account the finiteness of the electrical conductivity in the star, though very high, it is shown that the magnetic field of the star would gradually decay as a result of the Joule dissipation. The time for this decay is given by

$$\tau = \frac{4\pi \sigma}{c^2} l^2,$$

where $\sigma$ and $l$ are the electrical conductivity and the characteristic length of the star, respectively (e.g., Cowling, 1946, 1953). Now, let us consider this time for the Earth and Jupiter, for instance. If we assume that the electrical conductivity is $\sim 10^{12}$ esu in their interiors, the above times for these two planets are estimated as $\sim 10^4$ and $\sim 10^6$ yr, respectively. Hence these values are too short in comparison with their lives currently known. This means that the idea of 'fossil' magnetism is not applicable to explain the existence of the magnetism of the Earth and Jupiter. In order for the magnetism of the Earth, for instance, to be maintained, some mechanism to regenerate or amplify this magnetism must, therefore, be found in its interior.

As is well known, the maintenance of the Earth's magnetism is explained by considering the hydromagnetic self-exciting dynamo process in its interior. In this process, important is the existence of convective motion in the interior because this motion interact with the magnetic field lines ambient in the interior. At present, it is thought of that the origin of convective motion is closely related to the rotating motion of the Earth. This situation also seems to be applied to rotating magnetic stars as the Sun.

Since the Coriolis force must be taken into account in the study of rotating systems, we shall here investigate the effect of this force on the stability for the onset of convective motions inside rotating magnetic stars. Now, we first assume that the equilibrium state as given by Equations (1)-(3) is disturbed by applying a small positional perturbation $X$ to this state.

In this case, we obtain the equation of motion due to this perturbation as

$$q \frac{\partial^2 X}{\partial t^2} + 2\phi \Omega \times \frac{\partial X}{\partial t} = F(X)$$

and

$$F(X) = \text{grad} [\gamma P \text{div} X + (X \cdot \text{grad}) P] + j \times Q - B \times \text{curl} Q + \left[\text{div} (\varrho X)\right] \text{grad} \phi,$$

where $t$, $\phi$ and $\gamma$ are the time, the gravitational potential and the ratio of two specific heats, respectively, and $Q(X)$ is given by

$$Q(X) = \text{curl} (X \times B).$$

In the above equation, $\text{grad} \phi$ consists of two terms as the gravitational and the centrifugal forces, but, in general, the contribution of the centrifugal force is negligibly small (e.g., Chandrasekhar, 1961). In the cases which stars are rotating so fast as seen