NON-RADIAL OSCILLATIONS AND CONVECTIVE INSTABILITY OF A POLYTROPE WITH A TOROIDAL MAGNETIC FIELD

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Abstract. We examine the non-radial modes of oscillation, belonging to spherical harmonics of orders \( l = 1 \) and \( l = 3 \), of a gaseous polytrope with a toroidal magnetic field. We find that a toroidal magnetic field increases the growth rate of convective instability for deformations belonging to the spherical harmonic \( l = 1 \) whereas it decreases the growth rate of convective instability for deformations belonging to the harmonics \( l = 2 \) and \( l = 3 \). The frequencies of the 'acoustic' mode and the 'Kelvin' mode are decreased by the presence of the toroidal magnetic field.

1. Introduction

The non-radial modes of oscillation of a homogeneous compressible fluid sphere were first examined by Pekeris (1939). Chandrasekhar and Lebovitz (1963) used the virial method to study the non-radial oscillations of gaseous masses belonging to the spherical harmonics \( l = 1 \) and 3. They applied the theory to a polytropic gas sphere and determined the critical value of \( \gamma \) at which convective instability sets in. This value was found to be in excellent agreement with the value \( (1 + 1/n) \) obtained by an application of the Schwarzschild criterion. In a later paper, Chandrasekhar and Lebovitz (1964) studied the non-radial modes of oscillation of a polytrope belonging to the spherical harmonic \( l = 2 \), using a variational principle. Tassoul (1968) used the virial method to examine the non-radial \((l = 1\) and 3\)) modes of oscillation of a homogeneous gaseous sphere pervaded by a weak magnetic field of Prendergast type. He found that the magnetic field increases the frequencies of the acoustic mode belonging to the spherical harmonic \( l = 1 \) and of the Kelvin mode belonging to the spherical harmonic \( l = 3 \). Convective instability associated with the harmonic \( l = 1 \) can be suppressed only if \( \gamma \) is sufficiently large. For \( \gamma = \frac{4}{3} \) the growth rate of convective instability is actually increased by the presence of the magnetic field. The radial \((l = 0)\) and the non-radial \((l = 2)\) modes of oscillation of a polytrope with a toroidal magnetic field have been examined by Sood and Trehan (1972; this paper will hereafter be referred to as 'Paper I').

We now study the non-radial modes \((l = 1\) and 3\)) of oscillation of a magnetically distorted polytrope whose equilibrium has been discussed earlier in Paper I. The convective instability associated with the harmonic \( l = 1 \) is also examined.

2. The Formulation of the Problem

The linearized equations of motion governing small departures from the state of...
equilibrium can be written in the form

$$\sigma^2 q_\xi = T(\xi),$$

(1)

where $\xi(x,t) = \xi_0(x) \exp(i\sigma t)$ is the Lagrangian displacement applied to a fluid element located at $x$. If we let $\delta f$ denote the Eulerian change in the equilibrium value of $f$ due to the displacement $\xi$, the operator $T(\xi)$ can be written as

$$T(\xi) = \nabla \delta p - q \nabla \delta V - \frac{1}{c} \delta j \times H - \frac{1}{c} j \times \delta H - \frac{\delta q}{q} \left(\nabla p - \frac{1}{c} j \times H\right).$$

(2)

The Eulerian changes in the pressure, density, gravitational potential, magnetic field and current density are

$$\delta p = - \gamma p \nabla \cdot \xi - \xi \cdot \nabla p,$$

(3)

$$\delta q = - \nabla \cdot (q \xi),$$

(4)

$$\delta V = G \int \rho(x') \xi(x') \cdot \nabla |x - x'|^{-1} \, d\tau',$$

(5)

$$\delta H = \nabla \times (\xi \times H),$$

(6)

and

$$\delta j = \frac{c}{4\pi} \nabla \times \delta H.$$  

(7)

For the case under consideration, where the magnetic field vanishes at a boundary of the configuration, the variational principle can be written as (cf. Kovetz, 1966)

$$\sigma^2 \int \rho |\xi|^2 \, d\tau = \int \xi \cdot T(\xi) \, d\tau.$$  

(8)

It is convenient to scale the magnetic field terms by the quantity $h(=\lambda^2/4\pi^2 G)$ and split the operator $T(\xi)$ into two parts:

$$T(\xi) = -L(\xi) - hM(\xi),$$

(9)

where

$$L(\xi) = - \nabla \delta p + q \nabla \delta V + \frac{\delta q}{q} \nabla p$$

(10)

and

$$M(\xi) = \frac{1}{c} \left[ \delta j \times H + j \times \delta H - \frac{\delta q}{q} (j \times H) \right].$$

(11)

Clearly, $L(\xi)$ contains all the effects of a non-magnetic body, whereas $M(\xi)$ contains all the effects due to the magnetic field. Since the magnetic field has been regarded as