BULK VISCOSITY, KALUZA-KLEIN MODELS AND INFLATION

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Abstract. In this work we have employed two hypotheses which have been separately used in order to try to solve the horizon problem, the first one is to take a Kaluza–Klein cosmological model with $d$ non-compact and $D$ compact space-like dimensions, in particular we consider $D = 1$, the second one is to use an energy-momentum tensor depicting a fluid out of equilibrium, in particular we take a mixture of two gases, one is formed by relativistic particles and the other one is a gas constituted by non-relativistic particles and they are not in thermodynamical equilibrium, such that a bulk viscosity term arises. Without actually solving the Einstein equations, we prove that the scale factor of the non-compact space is a monotonic increasing function of time, and that if the scale factor of the compact space reaches a maximum at a certain time then the non-compact space is driven to expand rapidly, and, therefore, hinting us about the possibility of solving the horizon problem.

The effective pressure and density in the non-compact space are found and it is proved that they satisfy the condition for having generalized inflation, and, therefore, might permit to solve the horizon problem, even in the case of $D = 1$, there is no need of a large number of extra dimensions, as some other previous authors have found.

Despite our higher-dimensional matter is one in which the kinetic approach is valid, the effective tensor in the non-compact space-time has the property that this treatment is not applicable.

1. Introduction

Since the original work of Kaluza and Klein the interest in Kaluza–Klein (K–K) models has waned and waxed, but never has gone to zero. Besides been used to construct unified field theories of the fundamental interactions, K–K models have been employed to study higher-dimensional cosmologies (Chodos and Detweiler, 1980). The basic idea in these approaches is that the extra dimensions (usually called the compact space) are unobservable nowadays because they are compactified to a very small scale. In these cosmologies (Matzner and Mezzacappa, 1985), most of the effective equations of state in the non-compact space result to be radiation $p = \frac{1}{3} \rho$.

It has been suggested by Marciano that the time variation of fundamental constants might provide an evidence of the existence of extra dimensions.

Several authors, Alvarez and Gavela (1983) and Barr and Brown (1984), have claimed that the existence of extra dimensions could be held responsible for the large observed entropy of the Universe, and, therefore, these models might solve some old cosmological conundrums (Guth, 1981).

In this direction, there have been some attempts (Kolb et al., 1984; Abbott et al., 1984) in the quest of inflationary scenarios by means of a higher-dimensional Friedmann–Robertson–Walker (F–R–W) model, in these efforts, as was mentioned the background is a F–R–W metric, and the energy-momentum tensor is one associated

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with a higher-dimensional photon gas \( p = N^{-1} \rho \), where \( N \) is the total number of space-like dimensions, and it must be mentioned that the aforesaid tensor lacks any kind of dissipative terms. Though the flow in the \( N \)-dimensional space is isentropic, there is an increase of entropy in the non-compact space, that is a consequence of the compactification of the extra dimensions (Alvarez and Gavela, 1983; Barr and Brown, 1984) and as Guth (1981) pointed out, this fact may allow to solve the horizon problem. 

The most successful result in this direction (Abbott et al., 1984) solves the horizon problem only if there are forty or more extra dimensions, \( D \geq 40 \), being \( D \) the number of extra dimensions.

Concerning the search of inflationary scenarios, it has been claimed (Waga et al., 1986; Lima et al., 1988) that bulk viscosity, in a 4-dimensional space-time could drive inflation, but in this scheme the kinetic approach is non-valid.

In this article, we glue together these two ideas:

(a) a higher-dimensional space-time;

(b) an energy-momentum tensor that includes a non-vanishing bulk viscosity coefficient.

Our model is an \((N + 1)\)-dimensional space-time, there are \( d = 3 \) non-compact and \( D \) extra compact space-like dimensions, \( N = d + D \), our symmetry is \( R \sim S^d \times S^D \).

The line element is

\[
ds^2 = -dt^2 + r^2(t)g_{ij} \, dx^i \, dx^j + R^2(t)g_{ab} \, dx^a \, dx^b ,
\]

where \( i, j = 1, 2, 3 \); \( a, b = 3 + 1, 3 + 2, \ldots, 3 + D \); \( r \) and \( R \) are the scale factors of the \( d \) and \( D \) dimensional spaces, respectively and \( g_{ij} \) and \( g_{ab} \) are the maximally symmetric metrics of the corresponding spaces (Weinberg, 1972).

As some authors have stressed (Guth, 1981; Dicke and Peebles, 1979) the choice of a vanishing curvature is an unnatural assumption, hence we will take \(-1\) and \( k_D = 0, \pm 1 \) as the curvature of the \( d \) and \( D \)-dimensional spaces, respectively, the physically relevant value of \( k_D \) is \(+1\).

As has already been claimed (Diósi et al., 1984; Barrow, 1986), during and after the GUT phase transition the Universe was comprised of a mixture of highly relativistic and non-relativistic particles, with a mean interaction time of the order of the age of the Universe. Hence, the chosen energy-momentum tensor will be one depicting a \((3 + D)\)-space-like dimensional system constituted by a mixture of two gases: a relativistic gas and one formed by non-relativistic particles of mass \( m \), such that they are not in thermodynamical equilibrium. It is known, that this sort of systems present even in a 4-dimensional space-time a bulk viscosity term (Weinberg, 1971; Bernstein, 1988).

With the conditions set up above, we proceed in section 2 to prove that the scale factor of the non-compact space, \( r \) is a monotonic increasing function of time, also it will be shown that if \( R \) has a maximum, then for points near to \( R \sim 0 \), \( r \) is driven to expand rapidly.

The effective pressure and energy-density of the non-compact space will be found, and we are going to show that there is a region in time where the effective thermo-