THE LIGHT CURVED IN THE CM FIELD

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Abstract. In this paper we introduce the CM field in Sections 2 and 3 based on the paper by Wang and Peng (1985), and calculate the light curved in the CM field in Section 4. The result shows that P makes Δφ_CM larger than Δφ at θ ∈ (cos⁻¹ 3/2, π − cos⁻¹ 3/2), and smaller at θ ∈ (θ, cos⁻¹ 3/2) ∪ (π − cos⁻¹ 3/2, π). Under a special circumstance which source, CM lens, and observer are in the same line, if we get Δφ|θ=0 = 1/3 and Δφ|θ=π/2, we can determine the P(M) and Q(M) of the CM lens, M is the mass of the CM lens.

1. Introduction

The effects of the gravitational lens have been studied by many authors (cf. Einstein, 1936; Lieben, 1964; Young, 1981; Ibáñez, 1983; Schneider, 1987) but no people have discussed the effect of CM field lens, we think it is interesting to discuss it.

2. The Static Magnetic Field in the Background of the Magnetic R–N Metric

We discuss the static magnetic field in the static background. Below the Greek letters take the values of 0, 1, 2, 3; the signature is +, −, −, −; the Latin letters take the values of 1, 2, 3. Static space-time can be expressed as

\[ g_{\mu\nu} = g_{\mu\nu}(x^i) \, . \]

(2.1)

If a point in a 4-dimensional space-time moves along the time-line direction with the vector \( A \) remaining unchanged, the electromagnetic field

\[ F_{\mu\nu} = A_{\mu;\nu} - A_{\nu;\mu} \]

(2.1a)

is also static, the condition for it is that the Lie derivative equals zero in the form

\[ \xi^\mu A^\nu_{\cdot\mu} - A^\mu_{\cdot\nu} \xi_{\cdot\mu} = 0 \, , \]

(2.2)

where \( \xi^\mu \) is a time-like Killing vector satisfying the Killing equation

\[ \xi_{\mu;\nu} + \xi_{\nu;\mu} = 0 \, . \]

(2.2a)

The condition in which an electromagnetic field contains only pure magnetic field, can be expressed as

\[ F_{\mu\nu} = 2(\xi^\mu \xi_{\mu})^{-1/2} \xi_{\cdot\mu} B_\nu = 2g_{00}^{-1/2}(\xi_{\mu} B_\nu - \xi_{\nu} B_\mu) \, , \]

(2.3)

\[ \xi^0 = 1 \, , \quad \xi^i = 0 \, , \]

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where $B$ is a magnetic vector. We can consider $B$ as a pure magnetic space vector

$$B_\mu = -\frac{1}{2}(\xi^\mu \xi_\mu)^{-1/2} F_{\mu \nu} \xi^\nu, \quad (2.4)$$

because

$$F_{\mu \nu} \xi^\nu = (A_{\nu; \mu} - A_{\mu; \nu})_{;\mu} = (A_{\nu} \xi^\nu)_{;\mu} - (A_{\mu} \xi^\nu)_{;\nu} =$$

$$= A_0, \mu - (A_{\nu} \xi^\nu + A_{\mu} \xi^\nu), \quad (2.4a)$$

where $A = \xi^\mu A_\mu$. If we consider (2.2a), we can change the last term of (2.4) into

$$B_\mu = -\frac{1}{2}g_{00}^{-1/2} A_{0; \mu}, \quad (2.5)$$

From (2.3), we get

$$F_{\mu \nu} = \xi^\mu (g_{00}^{-1/2} B^\nu)_{;\nu} + (g_{00}^{-1/2} B^\nu \xi^\mu - \xi^\nu g_{00}^{-1/2} B^\nu - \xi^\nu g_{00}^{-1/2} B^\mu) -$$

$$- g_{00}^{-1/2} B^\mu \xi^\nu, \quad (2.6)$$

The last term in the above equation equals zero according to the Killing equation, the two terms in parentheses is the Lie derivative of $(-g_{00}^{-1/2} B^\nu)$ which is equal to zero because the field is static. So the above equation becomes

$$F_{\mu \nu} = \xi^\mu (g_{00}^{-1/2} B^\nu)_{;\nu}, \quad (2.6)$$

namely,

$$F_{\mu \nu} = \xi^\mu g_{00}^{-1/2} B^\nu. \quad (2.7)$$

Because of (2.7), the Maxwell equation in the empty space $F_{\mu \nu} = 0$ turns out to be

$$B^\prime_{ij} = \frac{1}{2}(g_{00}^{-1/2} g^{ij} A_{0, j})_{;i} = 0. \quad (2.8)$$

Wang (1978) and subsequently quoted literature give a static spherically-symmetric external metric of a celestial body with magnetic charge (for brevity hereafter referred to as)

$$d^2S = \left(1 - \frac{2GM}{c^2r} + \frac{Gq^2_m}{c^4r^2}\right) dx^2 - \left(1 - \frac{2GM}{cr} + \frac{Gq^2_m}{c^4r^2}\right)^{-1} dr^2 -$$

$$- r^2 (d\theta^2 + \sin^2 \theta d\phi). \quad (2.9)$$

Now we begin to search for the static magnetic field in the background magnetic R–N metric (2.9).

If we substitute (2.9) into (2.8), we get the equations satisfied by $A$ in the form

$$\left(1 - \frac{2m}{r} + \frac{kq^2_m}{r^2}\right) (r^2 A_{0, r})_{;r} + \frac{1}{\sin \theta} (\sin \theta A_{0, \theta})_{;\theta} +$$

$$+ \frac{1}{\sin^2 \theta} A_{\phi, \phi} = 0, \quad (2.10)$$