ANALYTICAL SOLUTION OF MAGNETOGASDYNAMIC CYLINDRICAL SHOCK WAVES IN SELF-GRAVITATING AND ROTATING GAS, II

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Abstract. Using the C.C.W. method, propagation of diverging cylindrical shock wave in a self-gravitating and rotating gas under the influence of a constant axial magnetic field has been studied for two cases of weak and strong shocks. Medium ahead of the shock is supposed to be homogeneous. Analytical relations for shock velocity and shock strength along with the expressions for the pressure, density, and particle velocity just behind the shock wave have been also obtained for both cases.

1. Introduction

Pai (1958, 1959) and Kumar et al. (1981) have investigated the propagation of hydromagnetic cylindrical shock wave through a self-gravitating gas. Kumar et al. (1982) and Kumar and Prakash (1982) have investigated the propagation of diverging cylindrical shock wave in an ideal gas in the presence of an axial magnetic field for both cases of weak and strong shock using the C.C.W. method (Chester, 1954; Chisnell, 1955, Whitham, 1958). The problem of propagation of diverging hydromagnetic cylindrical shock wave through a rotating gas have been solved by Kumar and Prakash (1983) using the C.C.W. method. Recently, Mishra and Singh (1987) have also discussed the propagation of diverging cylindrical shock waves through a rotating gas having an initial density and transverse magnetic field distribution variable.

The present paper reveals the problem of propagation of weak and strong diverging cylindrical shock waves through a self-gravitating gas having rotation in the presence of a constant axial magnetic field, using the C.C.W. method. The medium ahead of the shock is supposed homogeneous. The case of weak shock is investigated under two conditions: (i) when the magnetic field is strong and (ii) when it is weak. For strong shock also we have considered two cases: namely, (i) when the magnetic field is strong and (ii) when ratio of the densities on either side of the shock nearly equals to $(\gamma + 1)/\gamma - 1)$. In a later case the medium approximately become independent of the magnetic field.

At least we have obtained the expressions for the velocity, density, and pressure just behind the shock front.

2. Equations of Motion, Boundary Conditions, and Analytical Expressions for Shock Velocity

For cylindrical-symmetrical flow of a self-gravitating gas with rotation in the presence of an axial magnetic field, the equations of motion are

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{Gm}{r} - \frac{v^2}{\rho} + \frac{\mu H}{r} \frac{\partial H}{\partial r} = 0,
\]

\[
\left( \frac{\partial}{\partial t} + u \frac{\partial}{\partial r} \right) (uv) = 0,
\]

\[
\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} + \rho \left( \frac{\partial u}{\partial r} + \frac{u}{r} \right) = 0,
\]

\[
\frac{\partial H}{\partial t} + u \frac{\partial H}{\partial r} + H \frac{\partial u}{\partial r} + \frac{uH}{r} = 0,
\]

\[
\frac{\partial m}{\partial r} - 2\pi \rho r = 0,
\]

\[
\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} - a^2 \left( \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right) = 0;
\]

where \( r \) is the radial coordinate; \( u, v \) are the radial and azimuthal components of the particle velocity; \( m, p, \rho, H, \) and \( \mu \) denote, respectively, the mass inside a cylinder of radius \( r \), the pressure, the density, the axial magnetic field, and magnetic permeability of the gas. \( a = (\gamma p/\rho)^{1/2} \) is the sound velocity.

The magnetogasdynamics shock conditions can be written in terms of a single parameter \( N = \rho_1/\rho_0 \) as

\[
\rho_1 = N \rho_0, \quad H_1 = NH_0, \quad u_1 = \left( \frac{N - 1}{N} \right) U,
\]

\[
U^2 = \frac{2N}{(\gamma + 1) - (\gamma - 1)N} \left[ a_0^2 + \frac{b_0^2}{2} \left\{ (2 - \gamma)N + \gamma \right\} \right],
\]

\[
p_1 = p_0 + \frac{2\rho_0(N - 1)}{(\gamma + 1) - (\gamma - 1)N} \left\{ a_0^2 + \frac{\gamma - 1}{4} b_0^2(N - 1)^2 \right\},
\]

where \( 0 \) and \( 1 \), respectively, stands for the state immediately ahead and immediately behind the shock front; \( U \) is the shock velocity, \([a_0] \) is the sound speed \((\gamma p_0/\rho_0)^{1/2}\), and \( b_0 \) is the Alfvén speed \((\mu H_0^2/\rho_0)^{1/2}\).