ANGULAR RADIATION TRANSFER IN INHOMOGENEOUS DISPERSIVE MEDIA

E. A. SAAD¹, M. S. ABDEL KRIM², and A. A. EL GHAZALY¹

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Abstract. The equation of radiative transfer for an inhomogeneous dispersive finite medium subject to general boundary conditions is solved. The Padé approximation technique is used to calculate the angular distribution of radiation. Numerical results for the [0/1] Padé approximant lead to numerical results that compared with the exact results.

I. Basic Equations

Solution of the radiation transfer equation for obtaining the angular distribution of radiations in dispersive inhomogeneous media was considered by Özisik and Cengel (1984). In this work the Padé approximation technique is used to calculate the angular distribution of the outgoing radiation intensities \( I(x, \mu), I(x, -\mu) \) at the boundaries of a plane parallel slab of optical thickness \( a \) and radiation heat fluxes \( q^- (x), q^+ (x) \). Let us consider the equation of radiative transfer for an absorbing, emitting, isotropically-scattering inhomogeneous medium as

\[
\mu \frac{\partial}{\partial x} I(x, \mu) + I(x, \mu) = S(x) + \frac{1}{2} \lambda(x) \int_{-1}^{+1} I(x, \mu') d\mu',
\]

\[0 \leq x \leq a, \quad -1 \leq \mu \leq 1;\]

subject to the boundary conditions

\[
I(0, \mu) = f_1(\mu) + A_1 + \rho_1 J_1(\mu) + 2\rho_1^d \int_{-1}^{1} I(0, -\mu') d\mu', \quad \mu > 0, \tag{2}
\]

and

\[
I(a, -\mu) = f_2(\mu) + A_2 + \rho_2 J(\mu) + 2\rho_2^d \int_{-1}^{1} I(a, \mu') d\mu', \quad \mu > 0; \tag{3}
\]

where

\[
A_i = \varepsilon_i n^2 \sigma T_i^4 / \pi \quad (i = 1, 2), \quad J_1(\mu) = I(0, -\mu), \quad J_2(\mu) = I(a, \mu) \tag{4}
\]

and \( f_i(\mu) \), the intensity of externally-incident radiation; \( S(x) \), the external source; \( \varepsilon_i, \rho_i, \)

¹ Atomic Energy Authority, Nuclear Research Centre, Inchass, Egypt.
² Physics Department, Faculty of Science, Mansoura University, Damietta, Egypt.

\( \sigma, T_i \) are, respectively, emissivity, reflectivity, Stefan–Boltzmann constant, and surface temperature. Equation (1) is transformed to the integral equations

\[
I(x, \mu) = \exp(-x/\mu)I(0, \mu) + \frac{1}{\mu} \int_0^x \exp((x' - x)/\mu)g(x') \, dx'
\] (5)

and

\[
I(x, -\mu) = \exp(-(a - x)/\mu)I(a, \mu) + \frac{1}{\mu} \int_x^a \exp((x' - x)/\mu)g(x') \, dx',
\] (6)

where

\[
g(x) = S(x) + \frac{1}{2} \lambda(x) \int_{-1}^1 I(x, \mu) \, d\mu.
\] (7)

Inserting Equations (5) and (6) in Equation (7) we obtain

\[
g(x) = S(x) + \frac{1}{2} \lambda(x) \left\{ \int_{-1}^1 \exp(-(a - x)/\mu)I(a, -\mu) \, d\mu + \\
+ \int_0^1 \exp(-x/\mu)I(0, \mu) \, d\mu + \int_0^1 \int_0^1 E_1(|x - x'|) \, dx' \right\},
\] (8)

where

\[
E_n(x) = \int_0^1 \exp(-x/\mu)\mu^{n-2} \, d\mu
\] (9)

is the exponential integral function (cf. Abramowitz and Stegun, 1965).

Using the boundary conditions defined by Equations (2), (3), and (4) in Equation (8) we have

\[
g(x) = S(x) + \frac{1}{2} \lambda(x) \left\{ \int_{-1}^1 \exp(-(a - x)/\mu)I(a, -\mu) \, d\mu + \\
+ 2 \rho_2^d \int_0^1 I(a, \mu) \, d\mu + \int_0^1 \mu \exp(-x/\mu) \left[ f_1(\mu) + A_1 + \rho_2 J_1(\mu) + \\
+ \rho_1 J_1(\mu) \right] \, d\mu \right\}.
\] (10)

Equation (10), with Equations (5) and (6), gives the general form of the problem we going to study.