THE EFFECT OF FINITE ELECTRICAL CONDUCTIVITY ON
COMPRESSIBLE MAGNETOCONVECTION

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Abstract. The stability of a polytropic fluid layer in the presence of a uniform vertical magnetic field is studied under the combined influence of thermal and magnetic diffusion. The main objective of the present investigation is to examine the effect of finite electrical conductivity of the medium on the stability of hydromagnetic modes which are believed to be important in the sunspot phenomena. It is shown that the inclusion of finite electrical conductivity has destabilizing effect on convective modes and small-scale convection can occur in the presence of strong magnetic field, provided the magnetic diffusivity is larger than the thermal diffusivity. The magnetic diffusivity, however, has a tendency to stabilize the fast, slow, and Alfvén-modes.

1. Introduction

A variety of phenomena observed in the solar atmosphere are, in some way or the other, the outcome of the interplay between the magnetic field and the turbulent convective motion prevalent in the subphotospheric region of the Sun. Consequently, the study of interaction between convection and magnetic field, i.e., magnetoconvection, has received considerable attention in the astrophysical literature. Theoretical treatments of the problems of magnetoconvection deal almost exclusively with the Boussinesq approximation. Various aspects of magnetoconvection with possible applications to the solar atmosphere are discussed in monographs by Chandrasekhar (1961) and Cowling (1976) and also in reviews by Proctor and Weiss (1982) and Weiss (1985). The main advantage of the Boussinesq approximation is that it permits to explore neatly the effects of various dissipative processes in magnetoconvection and makes the physics of magnetoconvection more illuminating. For example, it has been shown (see Spruit, 1981) that the onset of convective instability in the presence of a uniform magnetic field depends critically on two factors:

(i) The ratio \( Q = \eta / \kappa \) of the magnetic diffusivity \( \eta \) (= \( C^2 / 4 \pi \sigma \)) to the thermal diffusivity \( \kappa \) (= \( K / \rho C_p \)), and (ii) the ratio \( S \) of buoyancy to Lorentz force; \( S = g x \beta 4 \pi \rho / k z B_0^2 \), in the usual notation. For closed boundaries, overturning convection occurs if \( S > 1 \). For \( S < 1 \), leak instability or overstable oscillations can occur. If \( \eta > \kappa \), the onset of instability occurs as leak instability due to magnetic diffusion, provided \( S > \kappa / \eta \). On the other hand, overstable oscillations can occur if \( \kappa > \eta \) and \( S > \eta / \kappa \). Thus, the Boussinesq calculations clearly demonstrate the role of the finite electrical conductivity of the medium on the onset of convective instability. Specifically, it tells us that leak instability cannot occur for \( S < 1 \), if the electrical conductivity of the medium is infinite.

The results of magnetoconvection have been used to explain certain striking features, e.g., umbral dots and umbral flashes in the sunspot umbra (see Beckers, 1981; and Moore, 1981). Bumba and Suda (1980) have suggested that umbral dots are of convective origin similar to normal photospheric granules. However, the observations of umbral dots by Kitai (1986) do not seem to favour the convective origin of umbral dots as suggested by Bumba and Suda (1980). Knobloch and Weiss (1984) studied the nonlinear magnetoconvection in the sunspot region to show that the umbral dots owe their origin to the oscillatory convection (Danielson, 1964) if the magnetic field extends below the umbra as a single coherent flux tube or monolithic plug (Meyer et al., 1974). Parker (1979a, b) has proposed a cluster or spaghetti model of the umbral magnetic field in which the magnetic flux splits into many isolated flux tubes separated by field-free region beneath the sunspot and umbral dots are then believed to be caused by intrusion of convective motions in the field-free region to the surface. Recent observations by Garcia de la Rosa (1987) seem to fit well into the penetrative convection model of the umbral dots proposed by Parker (1979b).

Rudraiah et al. (1985) have studied the two- and three-dimensional nonlinear magnetoconvection in a Boussinesq fluid subject to uniform vertical magnetic field. They have shown that overstable oscillations occur only for a restricted range \( (Q < 1) \) of the ratio \( Q \) of the magnetic to thermal diffusivity and overstable oscillations are likely to occur only in the upper layers of the sunspot umbra where \( \kappa > \eta \). Furthermore, they have argued that the umbral magnetic field as high as 3000 G is strong enough to favour oscillatory convection in the upper layers \( (\kappa > \eta) \) but are certainly subject to direct convection in the deeper layers \( (\eta > \kappa) \).

However, in the solar convection zone where density varies by several orders of magnitude, the Boussinesq approximation is of limited applications. In order to take into account the full effects of compressibility Antia and Chitre (1979) have studied the stability of a polytropic layer subject to a uniform vertical magnetic field in the presence thermal dissipation. In their work the magnetoacoustic or fast modes are found to be overstable and which can explain the properties of umbral flashes in the sunspot umbra. Similar calculations with uniform horizontal magnetic field (Antia et al., 1978) find applications to sunspot penumbral. In these investigations, however, the medium is assumed to have infinite electrical conductivity. This corresponds to \( Q = 0 \) and makes the stabilizing effect of the magnetic field more stringent. Furthermore, this assumption is not valid throughout the convection zone beneath the sunspot. In fact, \( \eta \) as well as \( \kappa \) vary strongly with depth in the sunspot region. In the upper region the value of the ratio \( Q = \eta/\kappa \) increases monotonically from a minimum value of \( \sim 10^{-5} \) to a value of unity at the depth of 2000 km and becomes as high as 40 at the depth \( \sim 20000 \) km (Meyer et al., 1974; Spruit, 1981; Knobloch and Weiss, 1984). Clearly, neglect of the effect of finite electrical conductivity in the upper 2000 km of the sunspot umbra can be considered as a good approximation. In this region overstable oscillations occur in the presence of an efficient thermal exchange process (Antia and Chitre, 1979). In the deeper region of the sunspot the effect of finite electrical conductivity of the medium should be given its due consideration. Owing to the magnetic diffusion stabilizing effect