A NOTE ON THE VALIDITY OF THE USE OF EULER’S EQUATION FOR THE MOTION OF THE MATTER IN THE ENVELOPES OF Be STARS

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Abstract. Three assumptions which underlie the theoretical treatment of the structure and kinematics of the circumstellar material surrounding Be stars are examined. It is shown that a single-fluid non-viscous hydrodynamical description is a reasonable one for describing the motion of a fluid under typical envelope conditions.

1. Introduction

Studies of the structure and kinematics of the envelopes of Be stars using Euler’s equation as the equation of motion have explained many of the phenomena associated with Be stars. Three very basic questions – Is the simple hydrodynamical approach used valid in the presence of ionized species? Is a single-fluid approach justified? Is it legitimate to ignore viscous terms in the equation of motion? – have received little discussion. In this note these three questions are briefly considered.

2. Applicability of the Hydrodynamic Equations for an Ideal Fluid

The hydrodynamical equations for an ideal fluid are derived from a Boltzmann transport equation whose ‘collision integral’ includes only binary collisions. In the presence of Coulomb interactions (and these are certainly present as the bulk of the hydrogen in Be star envelopes must be ionized), interaction involving three or more particles leading to the hydrodynamical equation for an ideal gas and the equations themselves are not strictly applicable if Coulomb effects play a role.

If the ideal-fluid hydrodynamical prescription is not generally valid in the presence of Coulomb forces among the particles, under what conditions should it be an adequate approximation? If the departure from a perfect gas is to be small (Zel’dovich and Razier, 1966) then (1) the Coulomb potential energy at mean separation distance \( e^2/n^{-3} \), where \( n \) is the particle density) should be much less than the average thermal energy \( (1.5 kT) \) and (2) the Debye length \( (kT/4\pi n e^2) \) must be much greater than the mean separation distance.

Here and in what follows, the following characteristic values will be assumed: stellar mass \( (M_\odot) \), 10 \( M_\odot \); stellar radius \( (R_\odot) \), 10 \( R_\odot \); stellar surface temperature \( (T_s) \), 25,000 K; equatorial rotational velocity \( (V_\phi) \), 344 km s\(^{-1}\); disk temperature \( (T_d) \), 10,000 K; molecular weight \( (\mu) \), 0.68; average disk density \( (\rho_d) \), \( 10^{-12} \) g cm\(^{-3}\).

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Assuming that all the hydrogen is ionized, the ratio of the Coulomb potential energy at mean separation to the average thermal length is $10^{-3}$, and the ratio of the Debye length to the mean separation is $8$. The first condition (1) above is clearly satisfied, and as complete ionization of the hydrogen is an idealization, the second, (2) above, is also satisfied.

3. Validity of a Single-Fluid Description

Changes in emission features characteristic of Be stars have been observed to occur in a time interval as small as a minute (Hutchings et al., 1972). Although such spectral variations do not necessarily imply variations in the flow variables over such a time scale, the validity of a simple single-fluid hydrodynamical description in the presence of flow variations over short times should be examined.

A single-fluid description is valid only so long as the relaxation time for appreciable energy transfer between any one species of the complex gas and the remaining species is less than the time scale over which variations in the flow occur. The relevant relaxation time for this problem is the time required for significant energy transfer between the electron component of the gas and the other species. It is sufficient to calculate the relaxation time for energy transfer between electrons and protons. This is given by Zel'dovich and Razier (1966, Equation 6-120) and is

$$\tau_{eh} = \frac{252T_e^{3/2}}{N_h \ln A}$$

where $N_h$ is the hydrogen-ion density and $A$ is the reduced Debye length. Under typical disk condition, $\tau_{eh} \approx 10^{-5}$ s. In the absence of any direct or indirect evidence for changes in the flow variables of such a short time scale, the use of a single-fluid description is justified.

4. Neglect of Viscosity in the Equation of Motion

If the viscous effects cannot be ignored, the equation of motion should be the Navier-Stokes equation. The importance of viscous effects relative to non-viscous ones can be determined by comparing the order of magnitude of the viscous terms in the Navier-Stokes equation to that of the non-viscous elements therein.

Denoting dimensional variable by a subscript $c$, a set of dimensionless variables is introduced (these carry no subscript). They are: density ($\varrho$), equal to $\varrho_c/\varrho_d$; radius vector ($r$), $r_c/R_c$; time ($t$), $t_cV_c/2\pi R_c$; velocity ($v$), $2\pi v_c/V_c$; pressure ($P$), $P_c (2\pi)^2/V_c\varrho_d$. Since $\varrho_d$, $R_c$ and $V_c$ are typical Be star values for their respective physical quantities, the dimensionless coordinates are of order unity. The Navier-Stokes equation is:

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{\nabla P}{\varrho} - \frac{4GM_s\pi^2}{R_s V_s^2} \left( -\frac{1}{r} \right) + \\
+ \frac{2\pi \eta}{R_s V_s \varrho_d} \frac{\nabla^2 \mathbf{v}}{\varrho} + \frac{2\pi(\xi + \eta/3)}{R_s V_s \varrho_d} \nabla(\nabla \cdot \mathbf{v}),$$