STABILITY VS INCLINATION OF MOTIONS IN $E^3$

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Abstract. Five families of three-dimensional doubly symmetric motions are computed after establishing their existence by means of a grid-search technique. It is confirmed that within the same family orbits of lower inclination with respect to the plane of motion of the primaries are stable while the critical inclination at which instability occurs varies between families. The maximum inclination at which stable motions of the type presented here were found is about 52°.

1. Introduction

The usual formulation of the restricted three-body problem for motions in three-dimensional space is adopted (see, e.g., Moulton, 1920) for the purpose of extending the existing limited output in closed motions. For the calculation of such motions we first use the method described by De Vogelaere (1958) through which, by suitable extension, we can locate such motions. Then a correction process is applied so that one member of each family can be computed. Once the first member is computed a procedure is applied in order to compute a sufficient number of members of the same family so that the family can be considered as known. This implies that all members used for the representation of each family are studied with respect to the properties that in some way are essential, e.g. the geometric form, the stability, inclination to the plane of the primaries, etc.

2. Procedure of Computation

The above described work was done for five families of closed solutions of double symmetry. Such solutions have been shown to exist (Goudas, 1961; Jefferys and Moser, 1966). Each of these solutions is symmetric with respect to the line through the two primaries and the plane that contains this line and the $Oz$-axis. The double symmetry of these solutions facilitates the calculations, since only one quarter of each solution suffices for the determination of all its properties. Indeed, as far as the geometrical figure of the solution is concerned a double reflection of one quarter of it can derive the entire solution. This also holds for the inclination of the orbit which is a geometric property and varies in a periodic fashion. On the other hand the calculation of the eigenvalues of the matrizant and hence the determination of the stability of the
solution can be done once the variational matrix for the closed solution is computed at the end of only one quarter of the period.

The procedure followed in the calculation is briefly described as follows: Once some member of each family is located by means of a grid-search method (for details, see Kazantzis and Goudas, 1974) a predictor-corrector-steepest descent algorithm is applied to compute a sufficient number of members of the same family. If the curve of initial conditions does not have points at infinity, as is the case in the five families presented here, then by moving along the curve in both directions and equal steps we can obtain a complete calculation of the family. Upon calculation of the initial condition of each member to a satisfactory accuracy, the programme then calculates the inclination of the orbit at its ascending mode. Since rotating coordinates where employed the inclination \( i \) was computed from the formula

\[
\tan i = \frac{x_{06}}{x_{01} + x_{05}},
\]

where the initial state \( x_0 \) of this type of solutions has the following components: \( x_{01}, 0, 0, 0, x_{05}, x_{06} \).

The stability of the orbit is the next property to be examined. This is done by calculating the quantities \( p \) and \( q \) where

\[
p + q = -[\text{Tr} A(x_0; T) - 2],
\]

\[
p \cdot q = \frac{1}{4} ([\text{Tr} A(x_0; T) - 2]^2 - [\text{Tr} A^2(x_0; T) - 2]) - 2;
\]

and \( A(x_0; T) \) is the matrizen of the orbit and \( T \) its period. When

\[
|p|, \quad |q| \leq 2
\]

the orbit is stable (Goudas, 1961), whereas when any of \( |p| \) and \( |q| \), or both are greater than two the orbit is unstable.

The relationship between stability and inclination is interesting, this is numerically investigated and presented for each family in separate diagrams given below.

### 3. Results

All the results obtained in connection with the five families of doubly-symmetric motions can be found in a Thesis by Halioulias (1974). In Table I below we list the initial conditions and the periods of four members of each family. The members presented rest on either side of the critical members of each family for which \( |p| = |q| = 2 \), i.e. the members at which stability changes. The dependence of \( p \) and \( q \) on the indication \( i \) permits the calculation of the critical inclination, i.e. the inclination of the critical orbits. The last column of Table I lists the inclination of the corresponding members of each family. The relationship between \( p \) and \( q \) and the inclination \( i \) for three out of the five families is presented in Figures 1 to 3 and covers Families I, II and III. The case of Family V is not presented in a diagram because the corresponding