EXACT BIANCHI TYPE-II, VIII, AND IX COSMOLOGICAL MODELS WITH MATTER AND ELECTROMAGNETIC FIELDS IN LYRA'S MANIFOLD

R. VENKATESWARLU and D. R. K. REDDY

Department of Applied Mathematics, Andhra University, Waltair, India

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Abstract. Spatially-homogeneous and anisotropic Bianchi type-II, VIII, and IX stiff-fluid cosmological models in the presence of source-free electromagnetic fields are obtained in Lyra's manifold. Some properties of the models are also discussed.

1. Introduction

Lyra (1951) proposed a modification of Riemannian geometry by introducing a gauge function into the structureless manifold, as a result of which the cosmological constant arises naturally from the geometry. Sen (1957) studied a static cosmological model in Lyra's manifold and others who have considered models based on this manifold geometry are Halford (1970), Bhamra (1974), and Kalyanshetti and Waghmode (1982).

It is well known that the simplest models of the expanding universe are spatially-homogeneous and isotropic. Beesham (1986a, b) has studied spatially-homogeneous and isotropic Friedmann–Robertson–Walker models in Lyra's manifold. However, in recent years, there has been considerable interest in spatially-homogeneous and anisotropic Bianchi cosmological models. The existence of anisotropy in such models allows a theoretical discussion of many important effects (Ryan and Shepley, 1975). Reddy and Innaiah (1985) have obtained a Bianchi type-I cosmological model in Lyra’s manifold. However, since Bianchi type-I models are a very special subset of spatially-homogeneous models, one should consider more general situations.

Lorenz (1980a, b) has presented exact Bianchi type-II, VIII, and IX stiff-fluid models in the presence of electromagnetic field in general relativity. In this paper we derive exact Bianchi type-II, VIII, and IX cosmological models with stiff fluid in the presence of electromagnetic field in Lyra’s manifold.

2. Metric and Field Equations

In an orthonormal frame the metric of space-time in the locally rotationally-symmetric case can be written (cf. Lorenz, 1980b) as

$$ds^2 = \eta_{ij} \sigma^i \sigma^j, \quad \eta_{ij} = \text{diag}(-1, 1, 1, 1); \quad (1)$$

where the certain basis \( \sigma^i \) is given by

$$\sigma^0 = dt, \quad \sigma^1 = R \omega^1, \quad \sigma^2 = R \omega^2, \quad \sigma^3 = S \omega^3. \quad (2)$$
Due to homogeneity the functions $R$ and $S$ depend only on $t$ and $\omega'$ are time-dependent one forms.

The field equations in normal gauge for Lyra's manifold as obtained by Sen (1957) are:

$$R_{ij} - \frac{1}{2} \eta_{ij} R + \frac{3}{2} \phi_i \phi_j - \frac{3}{4} \eta_{ij} \phi_k \phi^k = 8\pi (T_{ij} + E_{ij}), \quad (3)$$

$$E_{ij} = \frac{1}{4\pi} \left( F_{ik} F_{jl} - \frac{1}{4} \eta_{ij} F_{kl} F^{kl} \right), \quad (4)$$

$$T_{ij} = (\rho + p) u_i u_j + \eta_{ij} p, \quad (5)$$

where $R_{ij}$ is the Ricci tensor; $E_{ij}$, the electromagnetic stress energy tensor; $F_{ij}$, the Maxwell tensor; $T_{ij}$, the energy-momentum tensor; $\phi_i$, the displacement vector; $u^i$, the velocity 4-vector; and $\rho$ and $p$, the density and pressure of the fluid, respectively. We now assume the vector displacement field $\phi_i$ be the time constant vector

$$\phi_i = (\beta, 0, 0, 0), \quad \beta = \text{const}.\quad \text{(6)}$$

The source-free Maxwell equations are

$$dF = 0 = d\ast F, \quad (6)$$

where the two-form $F$ represents the electromagnetic field and $\ast F$ is its dual. Due to homogeneity, the electric field $F_{0i} = -E$ and the magnetic field $F_{23} = H$ depend only on $t$. Now the Maxwell equations (6) reduce to

$$(HR^2) - ES = 0$$

and

$$(ER^2) + HS = 0.\quad (7)$$

The solution of Equations (7) are given by

$$E = \frac{a}{R^2} \cos(t + b)$$

and

$$H = \frac{a}{R^2} \sin(t + b), \quad \text{(8)}$$

where $a$ and $b$ are constants.

Now the non-vanishing components of the electromagnetic stress-energy tensor are

$$E_{00} = E_{11} = E_{22} = -E_{33} = \frac{1}{8\pi} \frac{a^2}{R^4}. \quad (9)$$