THE OBSERVATIONAL DISCRIMINATION OF FRIEDMANN–LEMAÎTRE MODELS

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Abstract. It is the purpose of this paper to illustrate the interrelation between the problems of the 'missing mass', the galactic age and the cosmological constant \( \Lambda \) (or its equivalent quantum vacuum density \( \rho_v \)).

The inflationary picture of the early universe predicts that our present universe should have a very nearly Euclidean metric. If we accept this concept, one would have to discriminate between two rather extreme Euclidean cosmological models:

1. The standard model with \( \Lambda = 0 \) and a density \( \rho_c = 3H_0^2/8\pi G \). There are difficulties if \( H_0 \geq 50 \text{ km s}^{-1} \text{ Mpc}^{-1} \) and the galactic age \( t_0 \geq 14 \times 10^9 \) years.

2. The Euclidean Friedmann–Lemaître models with \( \Lambda > 0 \), i.e., \( \rho_v = \rho_c - \rho_0 \), where \( \rho_0 \) is the present matter density, including the nonrelativistic dark matter. Here \( \rho_v \) 'competes' with the missing mass.

Measurements of apparent diameters of galaxies up to redshifts of 2 will permit one to discriminate between the models provided that size evolution of galaxies can be determined or neglected (see Figure 3).

1. Introduction

In a recent paper Ehlers and Rindler (1987) pointed out that in principle it would be possible to determine the Friedmann–Robertson–Walker metric from observations of apparent luminosities \( (m(z)) \), angular sizes \( (\alpha(z)) \) and numbers \( (N(z)) \) of galaxies as function of redshift \( z \) provided that the evolution of the intrinsic luminosity of a standard galaxy \( (L(t)) \) and of the proper number density of galaxies \( (n(t)) \) as function of cosmic time \( t(z) \) or look-back time \( (t_0 - t(z)) \) would be known from theory. In view of the uncertainty of the presently available data, the most practical strategy would be the determination of the four parameters \( H_0 \) (Hubble number), curvature index \( k \), matter density \( \rho_0 \) at the present time \( t_0 \) and the cosmological constant \( \Lambda \) of Friedmann–Lemaître models (homogeneous-isotropic universe).

There are, however, essential difficulties not only with the theories of galactic evolution but also with selection effects, etc., in the observational data (see, for instance, Sievers et al., 1985).

On the other hand, the progress in the CCD-technique and the forthcoming launch of the Hubble Space Telescope make it likely that redshift observations of quasars and galaxies may be extended beyond \( z = 4 \).

Even with the present limit of \( z = 4 \) we are already able to look into the cosmic past
when the distances between galaxies were only 0.2 of the present ones:

\[ \frac{R(t(z))}{R_0} = \frac{1}{1 + z} = 0.2 \quad \text{for} \quad z = 4. \]

In Friedmann–Lemaître models a redshift of 4 can correspond to widely different cosmic times \( t(z) \) of the time of emission. We shall exemplify this with two somewhat extreme, but still realistic cases of models with Euclidean metric \((k = 0)\):

\[
\begin{align*}
H_0 &= 50 \text{ km s}^{-1} \text{ Mpc}^{-1}, \\
\rho_0 &= \rho_c = 3H_0^2/8\pi G = 4.7 \times 10^{-30} \text{ g cm}^{-3}, \\
\Lambda &= 0 \quad \text{or} \quad \rho_\nu = \Lambda c^2/8\pi G = 0, \\
t_0 &= 13.05 \times 10^9 \text{ years}, \\
t(z = 4) &= t_0(R(t)/R_0)^{3/2} = 1.2 \times 10^9 \text{ years}. \\
\end{align*}
\]

\[
\begin{align*}
H_0 &= 75 \text{ km s}^{-1} \text{ Mpc}^{-1}, \\
\Lambda &= 1.9 \times 10^{-56} \text{ cm}^{-2}, \\
\rho_0 &= 0.5 \times 10^{-30} \text{ g cm}^{-3}, \\
\rho_c &= 10.6 \times 10^{-30} \text{ g cm}^{-3}, \\
\rho_\nu &= \rho_c - \rho_0 = 10.1 \times 10^{-30} \text{ g cm}^{-3}, \\
t_0 &= 19.7 \times 10^9 \text{ years}, \\
t(z = 4) &= \tau \arccos \left( 1 + \frac{2\rho_\nu}{\rho_0} \left( \frac{R(t)}{R_0} \right)^3 \right) = 3.4 \times 10^9 \text{ years},
\end{align*}
\]

with

\[ \tau = 14.13 \times 10^9 \text{ years} (\rho_\nu(\_30))^{-1/2} = 4.4 \times 10^9 \text{ years} \]

(see Blome and Priester, 1985, and Figure 1).

In the second of these two Euclidean models the quasars at \( z = 4 \) have had an about three times longer evolutionary age: 3.4 versus \( 1.2 \times 10^9 \) years.

In cosmological papers one often finds the cosmological constant assumed to be zero. This neglects that Lovelock (1972) and Weinberg (1972) have shown that a strict derivation of Einstein's equation must contain a \( \Lambda \)-term. Thus it is a task for the observational cosmology to determine the value of \( \Lambda \). The \textit{a priori} restriction to models with \( \Lambda = 0 \) is not justified. The usual argument is that a \( \Lambda \)-term which is significant at our present epoch requires an unlikely fine-tuning. But in this argument it is overlooked that such a \( \Lambda \)-term most likely would have dominated the cosmic expansion for more than \( 10^{10} \) years, i.e., for most of the time of existence of the Universe.

A term \( \Lambda g_{\mu\nu} \) in Einstein's equation may be interpreted as the contribution of the