A NOTE ON THE PROBLEM OF CHOOSING A MODEL OF THE UNIVERSE, IV

(Letter to the Editor)

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Abstract. The Friedmannian model of the Universe with ultra-relativistic equation of state \( p = -\frac{1}{3} \rho \) with \( k = 0 \) is nondecelerative, i.e., it expands with the constant velocity \( v = c \) during the whole expanding phase of evolution of the Universe. The ultrastable nondecelerative model is the only model of the Universe which fulfills all conditions resulting from the quantum-mechanical and relativistic analysis of the Universe.

The Universe cannot be static. The first who encountered this fact was Einstein (1917) when constructing the first relativistic model of the Universe. That is to say, the equations of the general theory of relativity (GTR) do not allow to construct the static model of the Universe. The static state of the Universe was, however, in those times considered as the basic cosmological axiom. Einstein, faithful to the tradition, introduced into his cosmological equation so-called cosmologic member \( \lambda g_{\mu\nu} \) that should compensate the gravitational action of matter and radiation to obtain a static solution

\[
G_{\mu\nu} - \lambda g_{\mu\nu} = -\kappa (T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T) .
\]

However, it turned out later that Einstein’s cosmologic member cannot secure the static state of the Universe because Einstein’s static model of the Universe with the cosmologic member is extremely unstable. Any small failure that will occur in it would change it into a dynamic one (Eddington, 1930).

De Sitter (1917) created the first model of a dynamic expansive universe on the basis of Einstein’s model. The expansion of this model is the consequence of the increased effect of the cosmologic member.

Friedmann (1922, 1924) also derived his famous non-stationary models of the Universe from Einstein’s homogeneous and isotropic model of the Universe with hydrodynamic energy-momentum tensor. In contrast to Einstein and de Sitter who assumed a fixed value of the cosmologic member, Friedmann (1922) introduced the general assumption according to which the cosmologic member can acquire various values including \( \lambda g_{\mu\nu} = 0 \) in partial solutions.

The introduction of the general approach to the question of cosmologic member when applying the equations of GTR to the whole Universe was the first of the decisive steps to the definitive break with the traditional static conception of the Universe because it enabled Friedmann to write down Einstein’s and de Sitter’s models in the general form

\[
ds^2 = R^2 (dx_1^2 + \sin^2 x_1 \; dx_2^2 + \sin^2 x_1 \sin^2 x_2 \; dx_3^2) + M^2 \; dx_4^2 .
\]
Then Einstein’s model of stationary universe with the cosmologic member as the special partial case (2) can be expressed according to Friedmann in the form

$$d\tau^2 = -\frac{R}{c^2} (dx_1^2 + \sin^2 x_1 dx_2^2 + \sin^2 x_1 \sin^2 x_2 dx_3^2) + dx_4^2,$$  

and de Sitter’s model of the Universe as the special partial case (2) can be written as

$$d\tau^2 = -\frac{R^2}{c^2} (dx_1^2 + \sin^2 x_1 dx_2^2 + \sin^2 x_1 \sin^2 x_2 dx_3^2) + \cos^2 x_4 dx_4^2.$$  

The special result of Friedmann’s analysis was the description of a further – not considered till then – special partial case (2) with zero-value of cosmologic member – Friedmann’s (1922) nonstationary model of the Universe

$$ds^2 = R^2(x_4) (dx_1^2 + \sin^2 x_1 dx_2^2 + \sin^2 x_1 \sin^2 x_2 dx_3^2) + dx_4^2.$$  

In 1924 Friedmann published his second work dealing with the models of the Universe in which he specially analysed the possibility of the existence of the Universe with the negative curvature of space (Friedmann mentioned in his article that his friend professor Ya. D. Tamarkin drew his attention to the necessity of the special analysis of the possibility of a universe with the negative curvature).

Friedmann’s non-stationary solutions of the equations GTR for the whole of the Universe from the years 1922–1924 present the basis of principally new and the only correct cosmological conceptions (Monin et al., 1989). At present they are the mathematical and physical basis of the standard model of the Universe.

Today we already know that the non-stationarity of the Universe does not result only from GTR. Newtonian analysis of the homogeneous and isotropic models of the Universe leads to the results which agree with those of Friedmann (Milne, 1932; McCrea and Milne, 1934). These results do not change the meaning of the importance of Friedmann’s models for the development of cosmology.

1. Friedmannian Models of the Universe

At present there is a lot of literature dealing with Friedmann’s and Friedmannian models. We find there the Friedmannian metrics which can be expressed in various forms.

To determine gauge factor $a(\eta)$ and energy density $\varepsilon(\eta)$ expressed in dimensionless conform time $\eta$ defined by the relation

$$c \, dt = a \, d\eta,$$  

(6)