STABILITY OF THE COMPLETE CYCLE IN HYDROGEN COMBUSTION INSIDE THE STARS

XIANG-RU LI
Department of Physics, Jiangxi Normal University Nanchang, Jiangxi, China

and

FU-SHENG CHEN*
Department of Mathematics and Computer Science, Longwood College, Virginia, U.S.A.

(Received 13 July, 1990)

Abstract. In this paper, the stability of the complete cycle in hydrogen combustion inside the stars is investigated with generating-function and linear stability analysis methods based on the Prigogine's nonequilibrium thermodynamic theory (Nicolis and Prigogine, 1977).

1. Introduction

In view of the research developments, the descriptions of a series of nuclear combination processes come from the studies on abundance of elements. Most of the evolution processes observed on H–R map are determined by combustion stages of hydrogen and helium. Therefore, we investigate the stability of the complete cycle in hydrogen combustion with generating-function and linear stability analysis methods.

2. The Complete Cycle of Hydrogen Combustion Allowing for the Effect of Diffusion

The complete cycle in hydrogen combustion inside the stars is

\[ H^1 + H^1 \xrightarrow{k_1} D^2 + e^+ + \nu_e + 1.442 \sim 0.263 \text{ MeV}, \]
\[ D^2 + H^1 \xrightarrow{k_2} H^3_e + \gamma + 5.493 \text{ MeV}, \]
\[ H^3_e + H^3_e \xrightarrow{k_3} H^4_e + 2H^1 + 12.859 \text{ MeV}, \]
\[ H^3_e + H^4_e \xrightarrow{k_4} B^7_e + \gamma + 1.586 \text{ MeV}, \]
\[ B^7_e + H^1 \xrightarrow{k_5} B^8_e + \gamma + 0.135 \text{ MeV}, \]
\[ B^8_e \xrightarrow{k_6} B^8_e + e^+ + \nu_e + 17.98 \sim 7.2 \text{ MeV}, \]
\[ B^8_e \xrightarrow{k_7} 2H^4_e + 0.095 \text{ MeV}. \]

Considering that the speed of decay of $B^8_e$ and $B^8_e$ is much greater than that of the other reactions, for the convenience of calculation, we combine processes 5, 6, and 7.

* Permanent address: Department of Physics, Jiangxi University, Jiangxi, China.

and Equation (1) can be rewritten as
\[ H^1 + H^1 \xrightarrow{k_1} D^2 + e^+ + v_e + 1.442 \sim 0.263 \text{ MeV} , \]
\[ D^2 + H^1 \xrightarrow{k_2} H^3_e + \gamma + 5.493 \text{ MeV} , \]
\[ H^3_e + H^3_e \xrightarrow{k_3} H^4_e + 2H^1 + 12.859 \text{ MeV} , \]
\[ H^3_e + H^4_e \xrightarrow{k_4} B^7_e + \gamma + 1.586 \text{ MeV} , \]
\[ B^7_e + H^1 \xrightarrow{k_{567}} 2H^4_e + \gamma + e^+ + v_e + 18.210 \sim 7.2 \text{ MeV} . \]

Let \( A \) and \( B \) be the concentrations of the initial reactants \( H^1 \) and \( H^4_e \), which can be regarded as constants; \( X, Y, Z \) be the concentrations of the intermediates \( D^2, H^3_e \), and \( B^7_e \); and \( P, Q \) be the concentrations of the final products, respectively, then the following chemical reaction equations are obtained:
\[
\begin{align*}
A + A & \xrightarrow{k_1} X , \\
A + X & \xrightarrow{k_2} Y , \\
Y + Y & \xrightarrow{k_3} P , \\
B + Y & \xrightarrow{k_4} Z , \\
A + Z & \xrightarrow{k_{567}} Q .
\end{align*}
\]

The dynamical speed equations of the complete cycle in hydrogen combustion are
\[
\begin{align*}
\frac{\partial X}{\partial t} &= A^2 - AX + D_1 \nabla^2 X , \\
\frac{\partial Y}{\partial t} &= AX - qY^2 - BY + D_2 \nabla^2 Y , \\
\frac{\partial Z}{\partial t} &= BY - AsZ + D_3 \nabla^2 Z .
\end{align*}
\]

carried out the scale transformation
\[
X = \frac{k_2}{2} \tilde{X} , \quad Y = \frac{k_2}{2} \tilde{Y} , \quad Z = \frac{k_2}{2} \tilde{Z} ,
\]
\[
\tilde{t} = \frac{1}{k_1} \tilde{t} , \quad A = k_1 k_2 \tilde{A} , \quad B = k_1 k_4 \tilde{B} ,
\]
\[
D_i = k_1 \tilde{D}_i \ (i = 1, 2, 3) , \quad q = 8k_3/k_2^2 , \quad s = k_{567}/k_2 .
\]

Equation (4) can be rewritten as
\[
\begin{align*}
\frac{\partial X}{\partial \tilde{t}} &= A^2 - AX + D_1 \nabla^2 X , \\
\frac{\partial Y}{\partial \tilde{t}} &= AX - qY^2 - BY + D_2 \nabla^2 Y , \\
\frac{\partial Z}{\partial \tilde{t}} &= BY - AsZ + D_3 \nabla^2 Z .
\end{align*}
\]