GRAVITATIONAL INSTABILITY OF A HEAT-CONDUCTING PLASMA

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Abstract. The gravitational instability of an infinite, anisotropic, heat-conducting plasma is studied in this paper. It is found that, for the case of parallel propagation, the inclusion of heat-conduction terms in the fluid equations, in general, leads to overstability of the system, whereas the transverse propagation remains unaffected. We have solved numerically the dispersion relation corresponding to the parallel propagation and find that except for a range of wave numbers, the system is overstable. We also found that in the limit of vanishing zeroth-order heat flux, the condition for gravitational instability is similar to the Jeans's condition for instability for an isotropic plasma.

1. Introduction

The problem of gravitational hydromagnetic stability of an unbounded anisotropic plasma has been investigated by Tandon and Talwar (1963), Marochnik (1967), Kalra and Talwar (1970) and Kalra and Hosking (1970). These authors have used the Chew–Goldberger–Low (CGL) equations to study the gravitational instability of such a plasma. These equations, however, did not take into account, the effect of heat-conduction, which, as has been suggested by Whang (1971), becomes significant in situations, like that of the solar wind plasma. Huang et al. (1988) discussed the wave propagation and instability in such a heat-conducting anisotropic plasma and found that the inclusion of heat-conduction leads to the presence of two new wave modes in addition to the Alfvén, slow and fast magnetosonic modes. Kalra et al. (1985) have studied the firehose and mirror instabilities in a collisionless heat-conducting plasma in the limit of vanishing zeroth-order heat flux and found that the firehose condition remains unchanged, while the condition for mirror instability is modified.

We have studied here the effect of heat-conduction on the gravitational instability of an anisotropic plasma. It is found that the transverse propagation of waves in such a plasma is not affected by the inclusion of heat-conduction. For wave propagation, parallel to the magnetic field direction, we find that the firehose mode is unaffected, whereas the mode corresponding to the gravitational instability is modified. It is noted that, the effect of zeroth-order heat flux is in general, to make the system overstable except for a band of wave numbers, lying between two extremes, for which the system is stable. The lower end, as well as the upper end values of the band decrease with the increase of heat-conduction and further the band width becomes narrower with the increase of heat-conduction in the system.

2. Basic Equations

To include the heat-flux into the fluid equations, we, here use the Whang’s equations, which are obtained by using a special distribution function for collisionless, heat-conducting plasma. The distribution function, used by Whang (1971), may be written as \( f = [1 + c IJ h(c)]f_0 \), which is cylindrically-symmetric about the direction of the magnetic field, where \( f_0 \) is the usual bi-Maxwellian distribution function; \( c \), the intrinsic velocity; and \( h \), an even function of \( c \). Using this distribution function, it can be shown that, all even moments are independent of the function \( h \) and the fourth moment of \( f \) can be expressed as simple functions of lower moments. Thus no higher-order moment terms appear in the third-moment equation and a closed set of equations, including the heat-flux equations, is obtained.

These equations for a gravitating, anisotropic, heat-conducting, infinite plasma are written as:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{1}
\]

\[
\rho \frac{d\mathbf{v}}{dt} = -\nabla \cdot \mathbf{P} + \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B} + \rho \nabla \phi, \tag{2}
\]

\[
\frac{d}{dt} \left( \frac{B^2 P_\perp}{\rho^3} \right) = -2 \frac{B^3}{\rho^3} \mathbf{n} \cdot \nabla (q_\perp |B|), \tag{3}
\]

\[
\frac{d}{dt} \left( \frac{P_\perp}{\rho B} \right) = -\frac{1}{\rho} \mathbf{n} \cdot \nabla (q_\perp |B|), \tag{4}
\]

\[
\frac{d}{dt} \left( \frac{B^3 q_\parallel}{\rho^4} \right) = \frac{3}{2} \frac{B^2}{\rho^2} \mathbf{n} \cdot \left[ \frac{P_\parallel P_\perp}{\rho^2} \nabla B - B \frac{P_\parallel}{\rho} \nabla (p_\parallel /\rho) \right], \tag{5}
\]

\[
\frac{d}{dt} \left( \frac{q_\parallel}{\rho^2} \right) = \frac{1}{\rho B} \mathbf{n} \cdot \left[ \frac{P_\parallel^2}{\rho^2} \nabla B - B \frac{P_\parallel}{\rho} \nabla (p_\parallel /\rho) \right], \tag{6}
\]

where usual notations are used. The anisotropic pressure tensor is given by

\[
\mathbf{P} = P_\perp \mathbf{I} + (p_\parallel - p_\perp) \mathbf{n}\mathbf{n}, \tag{7}
\]

where \( \mathbf{n} = \mathbf{B}/|\mathbf{B}| \) and \( \mathbf{B} \) is the ambient magnetic field, which is uniform and is taken along the z-axis of the Cartesian coordinate system. The gravitational potential \( \phi \) satisfies the equation

\[
\nabla^2 \phi = -4\pi G \rho, \tag{8}
\]

Finally the equation, which couples the hydromagnetic flow vector \( \mathbf{v} \) with the magnetic field \( \mathbf{B} \) is given by

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}). \tag{9}
\]