GRAVITATIONAL SOURCES OF PURELY ELECTROMAGNETIC ORIGIN

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Abstract. The Einstein–Maxwell field equations for charged dust corresponding to static axially-symmetric metric of Levi-Civita have been studied. It has been shown that when the metric potentials \( g_\nu \) are functions of only one of the coordinates, viz., \( r \), the interior charged dust becomes purely of electromagnetic origin, in the sense that the physical quantities like the energy density, the effective gravitational mass, etc., are dependent only on the charge density and vanish when this charge density vanishes. Such models are known as electromagnetic mass models in the classical electrodynamics. An interior charged dust solution corresponding to this case has been obtained which, in a sense, represents an infinite dust distribution of electromagnetic origin. In the second case, viz., when the metric potentials are functions of the coordinates \( r \) and \( z \) both, it has been shown that some of the situations correspond to electromagnetic mass models. An example to illustrate this case has been obtained. This represents the source of the 'Reissner-Nordström-Curzon' field (an analogue of the Reissner-Nordström solution obtained by Curzon) which according to Curzon describes the exterior field of an electron.

1. Introduction

Of all the problems of classical electrodynamics, at the turn of this century (Feynman et al., 1964; Rohrlich, 1965; Jackson, 1975), the one that generated considerable interest was concerned with the structure of the electron. An extended electron, according to Lorentz, consists of only 'pure charge and no matter' and as such its total gravitational mass is purely of electromagnetic origin. Such models whose mass arises from the electromagnetic field alone are called electromagnetic mass models in the literature. Various attempts to explain and to find suitable mathematical representations of these models were made in the realm of classical electrodynamics, special theory of relativity and quantum mechanics. All such attempts were, however, found to be unsatisfactory in some respect or the other.

In a recent work, Tiwari et al. (1984) while studying the interior distribution of charged perfect fluid corresponding to the spherically-symmetric metric

\[
ds^2 = e^{\nu(r)} \, dt^2 - e^{\lambda(r)} \, dr^2 - r^2 \, (d\theta^2 + \sin^2 \theta \, d\phi^2)
\]

by imposing a relation \( g_{00} g_{11} = -1 \) on the metric (1.1), found that this condition ultimately leads to the invariant relation

\[
\rho + p = 0 \quad (\rho > 0, \, p < 0),
\]

where \( \rho \) and \( p \) are the proper energy density and the proper pressure, respectively. The equation of state (1.2), because of the pressure being negative, corresponds to a repulsive source of gravitation that describes spherically-symmetric electromagnetic mass models, in the sense that all the physical quantities like the proper energy density, the proper

pressure and the total gravitational mass depend only on the charge density of the electromagnetic field and vanish when the charge density vanishes. The problem has been further investigated by Gantreau (1985), Gron (1985, 1986), Tiwari et al. (1986), and Ponce de Leon (1987a, b, 1988).

We now address ourselves to the question whether we can dispense with the equation of state (1.2) representing a repulsive source to describe an electromagnetic mass model. That is, whether there exists an equation of state other than (1.2) representing an electromagnetic mass model which is not under tension. We have considered here a charged perfect fluid distribution filling the interior of a cylinder of finite radius corresponding to an axially-symmetric line element of Levi-Civita (1919). It is found (Section 2) that a charged perfect fluid source is compatible with the axially-symmetric field only when the pressure $p$ vanishes. It turns out that some of these axially-symmetric charged dust distributions correspond to electromagnetic mass models. The study has been divided into two parts. In the first part (Section 3) wherein the metric potentials are functions of the coordinate $r$ only, the charged dust distribution can exist only as of electromagnetic origin. However, the solution of this class, in a limiting sense, possess the character of line mass. In the second part (Section 4) wherein the metric potentials are functions of both the coordinates, viz., $r$ and $z$, the interior charged dust distribution corresponding to electromagnetic origin exists under certain conditions only. A solution has been obtained to illustrate this procedure. Interestingly, this solution turns out to be the source of the Curzon particle field. It may be mentioned here that the exterior field solution obtained by Curzon (1924) after transforming to Schwarzschild coordinates, reduces to the Reissner–Nordström solution wherein the constant of integration associated with the mass parameter is purely dependent on electrostatic potential. It vanishes when the electrostatic potential vanishes, i.e., the Curzon field reduces to a flat space. One may alternatively say that the mass of the exterior Curzon solution is purely of electromagnetic origin. Curzon, therefore, called the solution thus obtained as the exterior field of an isolated electron. This only means that he believed in the definition of the electron of Lorentz as ‘pure charge and no matter’. The interior solution obtained in this paper gives the source of the so-called Reissner–Nordström–Curzon field, which hitherto is unknown.

2. Reduction of Axially-Symmetric Charged Perfect Fluid to Charged Dust Distribution

We consider an axially-symmetric Levi-Civita’s (1919) static metric in the form

$$ds^2 = g_{ij} dx^i dx^j = e^{2\beta} dt^2 - e^{-2\alpha} (dr^2 + dz^2) - r^2 e^{-2\beta} d\phi^2,$$

where $\alpha$ and $\beta$ are functions of the coordinates $r$ and $z$ only. The coordinates $x^0, x^1, x^2, \text{ and } x^3$ correspond to $t, r, z, \text{ and } \phi$, respectively.

$$\begin{align*}
\end{align*}$$