FINITE LARMOR RADIUS AND HALL EFFECTS ON
THERMOSOLUTAL INSTABILITY OF A ROTATING PLASMA

K. C. SHARMA and URVASHI GUPTA
Department of Mathematics, Himachal Pradesh University, Shimla, India

(Received 22 May, 1990)

Abstract. Thermosolutal instability of a rotating plasma with finite Larmor radius and Hall effects is studied. When the instability sets in as stationary convection, the Hall currents and the stable solute gradient are found to have destabilizing and stabilizing effects, respectively. For the case of no rotation, finite Larmor radius effects are always stabilizing for \( x = (k^2d^2/\pi^2) \) greater than two and for \( x \) less than two they have a stabilizing or destabilizing influence depending on whether \( N( = \nu_0/\nu) \) is greater than or less than its critical value \( N_\star \). In the limit of vanishing Hall current, the stabilizing effect of Coriolis force is observed. The question of onset of instability as overstability is also discussed.

1. Introduction

The effects of the finite Larmor radius, which exhibits itself in the form of a magnetic viscosity in the fluid equations, on plasma instabilities have been studied by Rosenbluth et al. (1962), Roberts and Taylor (1962), and Jukes (1964). The problem of thermal instability in the rotating layer of a conducting fluid in the presence of a uniform vertical magnetic field has been discussed by Chandrasekhar (1961). Veronis (1965) has investigated the problem of thermohaline convection in a horizontal layer of viscous fluid heated from below whereas the problem of thermohaline convection in a horizontal layer of viscous fluid, heated from below and salted from above has been studied by Nield (1967). Sharma and Sharma (1981) have studied the effect of finite Larmor radius on thermosolutal instability of a plasma. The conditions under which convective motions are important in stellar atmospheres are usually far removed from consideration of a single component fluid and rigid boundaries, and, therefore, it is desirable to consider a fluid acted on by a solute gradient and free boundaries. Under such conditions, buoyancy forces can arise not only from density differences due to variations in temperature, but also from those due to variations in solute concentration.

In the present paper we investigate the finite Larmor radius and Hall effects on thermosolutal instability when the plasma layer is subjected to uniform rotation and permeated by a uniform vertical magnetic field. The plasma is considered in the form of an infinite horizontal layer of thickness \( d \) heated from below and subjected to a stable solute gradient so that the temperatures and concentrations at the bottom surface \( z = 0 \) are \( T_0 \) and \( C_0 \) and at the upper surface \( z = d \) are \( T_1 \) and \( C_1 \), respectively, the \( z \)-axis being taken as vertical. This layer is in a state of uniform rotation \( \Omega(0, 0, \Omega) \) acted on by a uniform vertical magnetic field \( \mathbf{H}(0, 0, H) \) and a gravity force \( \mathbf{g}(0, 0, -g) \). This layer is heated from below such that a uniform temperature gradient \( \beta( = |dT/dz|) \) and uniform solute gradient \( \beta' ( = |dC/dz|) \) are maintained.

2. Perturbation Equations

Let $\rho, T, C, \alpha, \alpha', g, \mathbf{q}(u, v, w), \mu, \nu, \kappa, \kappa', \eta, \gamma, N', \delta \rho, p, e$ denote, respectively, the density, temperature, solute concentration, thermal coefficient of expansion, an analogous solvent coefficient of expansion, gravitational acceleration, velocity, magnetic permeability, kinematic viscosity, thermal diffusivity, solute diffusivity, resistivity, the stress tensor, number density, and the charge of an electron. Using the Boussinesq approximation, the equations expressing the conservation of momentum, mass, temperature, solute mass concentration, and equation of state are

\[
\frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q} \cdot \nabla) \mathbf{q} = -\frac{1}{\rho_0} \nabla p + \nu \nabla^2 \mathbf{q} + g \left( 1 + \frac{\delta \rho}{\rho_0} \right) + \frac{\mu_e}{4\pi \rho_0} (\nabla \times \mathbf{H}) \times \mathbf{H} + 2(\mathbf{q} \times \mathbf{Q}),
\]

(1)

\[
\nabla \cdot \mathbf{q} = 0,
\]

(2)

\[
\frac{\partial T}{\partial t} + (\mathbf{q} \cdot \nabla) T = \kappa \nabla^2 T,
\]

(3)

\[
\frac{\partial C}{\partial t} + (\mathbf{q} \cdot \nabla) C = \kappa' \nabla^2 C,
\]

(4)

\[
\rho = \rho_0 [1 - \alpha(T - T_0) + \alpha'(C - C_0)],
\]

(5)

where the zero suffix refers to the values at the reference level $z = 0$.

Maxwell's equations give

\[
\frac{d\mathbf{H}}{dt} = (\mathbf{H} \cdot \nabla) \mathbf{q} + \eta \nabla^2 \mathbf{H} - \frac{1}{4\pi N} \nabla \times [(\nabla \times \mathbf{H}) \times \mathbf{H}],
\]

(6)

\[
\nabla \cdot \mathbf{H} = 0,
\]

(7)

where $d/dt = \partial/\partial t + \mathbf{q} \cdot \nabla$ stands for the convective derivative.

The steady-state solution is

\[
\mathbf{q} = 0, \quad T = T_0 - \beta z, \quad c = c_0 - \beta' z, \quad \rho = \rho_0 (1 + \alpha \beta z - \alpha' \beta' z).
\]

(8)

Consider a small perturbation on the steady-state solution and let $\mathbf{q}(u, v, w), \delta p, \delta \rho, \theta, \gamma,$ and $\mathbf{h}(h_x, h_y, h_z)$ denote, respectively, the perturbations in velocity, pressure, density, temperature, concentration, and magnetic field. The change in density $\delta \rho$, caused by the perturbations $\theta$ and $\gamma$ in temperature and concentration, is given by

\[
\delta \rho = -\rho_0 (\alpha \theta - \alpha' \gamma).
\]

(9)