FLUCTUATIONS IN INTERSTELLAR GRAIN TEMPERATURES

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Abstract. An analysis has been made of temperature fluctuations expected for various grain materials exposed to the interstellar radiation field. It is shown that in all but the densest clouds average grain temperatures in the 5–10K range are of little statistical significance because of large fluctuations produced by absorbed photons from the interstellar radiation field. At higher average temperatures large fluctuations may still be present for certain grain materials and grain radii. The effect of these fluctuations on the simple problem of H atom recombination on grain surfaces is discussed.

As interstellar grains are known to play an important role in the formation of interstellar molecules, the temperature of these grains is of fundamental significance. Recently there have been a number of calculations of grain temperature for a variety of grain models and under a variety of conditions to be expected in the interstellar medium (Werner and Salpeter, 1969; Purcell, 1969; Greenberg and de Jong, 1969; Field, 1969). In these calculations the grain temperature $T_g$ is obtained as the solution to the energy balance equation

$$\int_0^\infty J_\lambda Q_\lambda d\lambda = \int_0^\infty B_\lambda(T_g) Q_\lambda d\lambda , \quad (1)$$

where $J_\lambda$ is the interstellar radiation flux, $Q_\lambda$ is the absorption efficiency and $B_\lambda(T_g)$ is the blackbody radiative flux for a surface at temperature $T_g$. Since $J_\lambda$ can be taken to be of the form (Habing, 1968)

$$J_\lambda = \sum_i W_i B_\lambda(T_i) , \quad (2)$$

where $W_i$ are dilution factors for blackbody sources at temperature $T_i$ with $T_i \approx 10^4$ K, the first terms in Equation (1) derives its value mainly from wavelengths in the ultraviolet, visible and near infrared regions of the spectrum. The second term in Equation (1) covers wavelengths which lie mainly in the far infrared because of the relatively low values which $T_g$ assumes.

In the present article we will examine the extent to which this time-averaged temperature can actually be taken to characterize the fluctuating temperature of grains exposed to a dilute radiation field. It will be shown that for likely grain materials, the time dependent grain temperature $T(t)$ fluctuates greatly for certain ranges of $r$, the grain radius and $T_g$ the time-averaged temperature given by Equation (1). These fluctuations further complicate calculations of molecule formation on grain surfaces but must be considered in future studies of recombination kinetics.

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In estimating temperature fluctuations we will assume that grains have not yet accreted mantles by condensation of heavy atoms and molecules from the interstellar gas. Grains will therefore be taken to be spherical refractory particles with radii in the range $10^{-6}$ cm $\leq T \leq 10^{-5}$ cm. Calculations will be performed for a variety of likely grain materials.

If we denote the internal energy of a grain measured relative to its zero-point energy as $U$ then

$$\Delta U = C_v(T) \Delta T$$

is the relation between a change in this internal energy and the corresponding temperature fluctuation $\Delta T$. $C_v(T)$ is the temperature dependent heat capacity of the grain. If such a grain is now placed in a blackbody cavity at temperature $T$, the mean squared fluctuation in $U$ is

$$\overline{\Delta U^2} = C_v(T) kT^2,$$

where we assume that the particle has many degrees of freedom (Tolman, 1938). Using Equation (3) the mean squared temperature fluctuation is

$$\overline{\Delta T^2} = \frac{kT^2}{C_v(T)}.$$  

This fluctuation arises because of randomness in the emission and absorption of photons by the particle.

In the interstellar medium a particle is subjected to a radiation field which is the superposition of several blackbody fields. The grain is also emitting radiation to maintain its average temperature $T_0$. Equation (5) therefore grossly underestimates the mean-squared fluctuation in $T$ that will be experienced by a grain heating and cooling in this way. This equation is expected to provide only an estimate of the order of magnitude of $\overline{\Delta T^2}$ produced by the emission of infrared photons by the grain i.e. fluctuations corresponding to the second term in Equation (1).

From Smith et al. (1957) one can express the mean-squared fluctuation in a flux of radiative energy $F_\nu$ for bandwidth $\Delta f$ as

$$\overline{\Delta F^2} = 2\Delta f \int_0^\infty \frac{F_\nu h\nu e^{h\nu/kT}}{[e^{h\nu/kT} - 1]} d\nu.$$  

To obtain the fluctuation $\overline{\Delta J^2}$ we must evaluate the summation

$$\overline{\Delta J^2} = 2\Delta f \sum_i W_i \int_0^\infty \frac{B_v(T_i) h\nu e^{h\nu/kT_i}}{[e^{h\nu/kT_i} - 1]} d\nu,$$

which reduces to

$$\overline{\Delta J^2} = 8\sigma k\Delta f \sum_i W_i T_i^5,$$