CURVATURE EFFECTS IN EXTENDED STELLAR ATMOSPHERES – ABSORPTION AND SCATTERING

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Abstract. The numerical solution of radiative transfer equation including curvature with both absorption and scattering has been developed in the framework of Discrete Space Theory. Two cases have been considered: (A) irradiation of the atmosphere at \( r = T \) and (B) no irradiation on either side of the atmosphere. Isotropic scattering has been assumed. It is found that the emergent luminosities (defined by \( \int r^2 I(r, \mu) \, d\mu \)) from scattering dominated atmospheres are smaller than those from absorption dominated atmospheres.

1. Introduction

There are several ways of solving the radiative transfer equation, among which the discrete ordinate method is well-known (Chandrasekhar, 1950). This cannot be extended, however, to a non-uniform medium and especially for curved geometries. From the work of Carlson (1963) and Lathrop and Carlson (1967), one can construct difference equations (that conserve energy) from the transfer equation for quite general non-uniform media and for arbitrary curvilinear co-ordinate system. The solution can then be obtained by an iterative procedure, while maintaining stability, for a medium that both absorbs and scatters radiation. The iteration converges fairly well whenever the optical thickness of the medium is not too high and the albedo for single scattering is not too close to unity. Otherwise, the iteration fails to converge properly and we cannot guarantee a good solution. Discretization errors, furthermore, are rather serious.

However, it was Grant (1968) who first analysed systematically these techniques and made attempts not only to overcome the difficulties of the discretization errors but also the instability situations (particularly for large optical thicknesses and for albedo for single scattering equal to unity) by rewriting the equations in what is called the ‘invariant \( S_\nu \)’ form (Grant and Hunt, 1968, 1969a, b), for the Carlson’s difference equations are known to be conservative and the principles of invariance are a natural property of the reflection and transmission operators associated with subdivisions of the medium of interest. The matrix structure allows the desired analysis of the system of equations and enables us to construct a simple direct method of calculating the solution. In this way we no longer need to iterate the solution for scattering problems, and at the same time maintain stability of the solution throughout the radiation field.

This theory was extended to the problems of spherical geometry (Peraiah and Grant, 1973; Peraiah, 1973a, henceforth called Paper I and Paper II, respectively) and to formation of spectral lines in Non-LTE atmospheres (Grant and Peraiah, 1972; Peraiah, 1973b). In Paper I, we have investigated a purely isotropic conservative scat-
tering atmosphere and found that the energy is conserved to the machine accuracy. In Paper II, atmospheres with pure absorption have been investigated. However, these are rare in practice and they have been studied from a purely academic interest and to provide guidelines for studying the atmospheres with both absorption and scattering, which we shall study in this paper.

In Section 2, we briefly outline the procedure for obtaining the solution and discuss the results in Section 3.

2. A Brief Description of the Method

The equation of radiative transfer in spherical symmetry is written as,

\[ \frac{\mu}{r^2} \frac{\partial}{\partial r} \{ r^2 I(r, \mu) \} + \frac{1}{r} \frac{\partial}{\partial \mu} \{ (1 - \mu^2) I(r, \mu) \} \]

\[ + \sigma(r) I(r, \mu) = \sigma(r) \left\{ [1 - \omega(r)] b(r) + \int_{-1}^{+1} p(r; \mu, \mu') I(r; \mu, \mu') d\mu' \right\} , \]

where \( I(r, \mu) \) is the specific intensity, \( r \) the radius, \( \mu = \cos \theta \), \( \sigma(r) \) the absorption coefficient \( b(r) \) the internal source and \( \omega(r) \) is the albedo for single scattering (\( \omega(r) = 0 \) for pure absorption and \( \omega(r) = 1 \) for pure scattering) subject to the conditions that

\[ \sigma(r) \geq 0, \]

\[ b(r) \geq 0, \]

\[ 0 \leq \omega(r) \leq 1, \]

and

\[ 0 < r \leq r_{N+1} ; \]

\[ p(r; \mu, \mu') \] is the scattering phase function, normalized so that,

\[ \frac{1}{2} \int_{-1}^{+1} p(r; \mu, \mu') d\mu' = 1, \]

and

\[ p(r; \mu, \mu') \geq 0, \]

with

\[ -1 \leq \mu \leq 1. \]

The discrete equivalent of (3) is written following Grant and Hunt (1968) as

\[ \frac{1}{2} \sum_{j=1}^{m} \left( p_{ij}^{++} + p_{ij}^{--} \right) c_j = 1, \quad 1 \leq j \leq m ; \]

\[ \frac{1}{2} \sum_{i=1}^{m} c_i \left( p_{ij}^{++} + p_{ij}^{--} \right) = 1, \quad 1 \leq j \leq m ; \]