TWO-COMPONENT, CONCENTRIC, AND CO-POLAR HOMOGENEOUS SPHEROIDS IN VIRIAL EQUILIBRIUM: A REVIEW WITH ADDITIONAL RESULTS

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Abstract. The theory of two-component, concentric, and co-polar homogeneous spheroids in virial equilibrium is reviewed and extended, with explicit expressions of some physical parameters and a detailed analysis of a number of limiting situations. First a fixed configuration with given physical parameters is taken into consideration, then further investigation is devoted to configurations moving along a specified sequence. As a possible application of the theory, galaxies are modelled as two-component spheroids of the kind considered and further analysis is performed, which allows a description of galactic evolution characterized by contraction (via energy dissipation) of one component and mass transfer (via star formation) from the contracting component to the other. The results of the current paper are to be intended as an organic and complete reference, that might help additional work involving any generalization of the method and/or application of the theory.

1. Introduction

Concentric and co-polar homogeneous spheroids make the simplest choice – among aspherical, two-component systems in virial equilibrium – to represent real galaxies. Though substantial work has been performed concerning a fixed configuration (e.g., Brosche et al., 1983; Caimmi et al., 1984), an additional effort has to be devoted on these topics in dealing with evolution along a specified sequence (e.g., Caimmi and Secco, 1990). In fact, explicit expressions of some physical parameters and a detailed analysis of a number of limiting situations, are needed in order to get a better understanding on the problem under consideration. The present attempt is intended to yield nothing essentially new on the method, but, on the contrary, to review and extend all the results which might be quoted in future work involving any generalization and/or application of interest.

In general, we limit our attention to a very simple case – namely: (i) the subsystems are concentric and co-polar (i.e., with parallel polar axes; for an extension to concentric and co-axial systems, see Caimmi et al., 1984); (ii) one subsystem lies completely within the other; (iii) each subsystem rotates around the polar axis (not necessarily at the same rate); (iv) the virial theorem (in tensor form) holds. In consequence of (i) and (iii), there is isotropy of both peculiar and rotational velocity on the equatorial plane, for each subsystem. It is noteworthy that a given kinetic energy tensor corresponds to infinite pairs of rotational and peculiar velocity distributions.

In Section 2 the theory of two-component, concentric, and co-polar homogeneous spheroids in virial equilibrium is reviewed and extended concerning a fixed con-
configuration, while the same is done in Section 3 in dealing with evolution along a specified sequence. As a possible application of the theory, galaxies are modelled as two-component spheroids in Section 4, and further analysis is performed. Finally, some concluding remarks are reported in Section 5.

2. Fixed Configurations

Let $i$ denote the inner subsystem and $j$ the outer, $O$ be the common center, $Oxy$ the equatorial plane, and $z$ the polar axis, and choose $(Oxyz)$ as frame of reference. Under the simple assumptions made in Section 1, the components of the (half potential, or kinetic) energy tensor corresponding to the coordinate pair $x_p x_q$ of the two subsystems is found to be (e.g., Caimmi and Secco, 1990):

$$ (E_u)_{pq} = -\frac{3}{10} \frac{GM_u^2}{a_u} (K_{uv})_{pq}, \quad (1) $$

$p = x, y, z$, $q = x, y, z$, $u = i, j$, $v = j, i$;

with $G$ constant of gravitation, $M_u$ and $a_u$ mass and equatorial semi-axis of the $u$th subsystem, and $E_u$ half-potential energy or, when the virial theorem holds, total energy of the $u$th subsystem. The quantities $(K_{uv})_{pq}$ are defined according to (e.g., Caimmi and Secco, 1990):

\[
(K_y)_{xx} = (K_y)_{yy} = \frac{1}{2} \left[ \frac{x_i + m x_j}{y^3} \right],
\]

\[
(K_y)_{zz} = \frac{1}{y_i} \left[ \frac{y_j}{y^3} \right],
\]

\[
(K_{ji})_{xx} = (K_{ji})_{yy} = \frac{1}{2} \left[ \frac{x_j}{y^3} \right],
\]

\[
(K_{ji})_{zz} = \frac{1}{y_j} \left[ \frac{x_i}{y^3} \right],
\]

\[
(K_y)_{pq} = (K_{ji})_{pq} = 0, \quad p \neq q;
\]

where $m$, $y$, and $\eta$ denote outer/inner ratios of masses, equatorial axes, and polar/equatorial ratios of axes (indicated by $\varepsilon$):

\[
m = \frac{M_j}{M_i}, \quad y = \frac{a_j}{a_i}, \quad \eta = \frac{\varepsilon_j}{\varepsilon_i}, \quad \varepsilon_u = \frac{c_u}{a_u}
\]

and, in addition

\[
\phi = \frac{E_j}{E_i} = \frac{m^2}{y} \frac{2(K_y)_{xx} + (K_{ji})_{zz}}{2(K_y)_{xx} + (K_y)_{zz}}
\]