ON THE ROCHE CONSTANTS FOR MAIN-SEQUENCE BINARIES

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Abstract. The ratios \( C_1/C_2 \) of the constants defining the equipotential surfaces which describe the external forms of the components of a close binary system have been calculated on the basis of evolutionary models. Theoretical systems have been considered allowing for a wide range of input parameters (masses and separation) and taking into account the evolutionary effects on the radii of the stars during their Main-Sequence lifetime. The systems have not undergone any transfer of matter and are representative of detached binaries with Main-Sequence components. The ratios of the constants are confined in limited intervals and, for the highest values of the mass-ratio, they are clustered around the unit.

1. Introduction

Equipotential surfaces which describe the external forms of either component in a close binary system can be represented (cf. Kopal, 1959; p. 481) by the expression

\[
G = 2(1 - \mu_j) + \frac{2\mu_j}{\sqrt{1 + r_j^2}} + r_j^2 + \mu_j^2,
\]

where \( C_j (j = 1, 2) \) is a constant; \( r_j \), the fractional radius of either star; and

\[
\mu = \frac{m_3 - 1}{m_1 + m_2}.
\]

The system of coordinates has its origin at the centre of gravity of the more massive star \( m_1 \); the coordinate frame rotates so that the star \( m_2 \) lies always on the \( x \)-axis while the \( z \)-axis is perpendicular to the plane of the orbit. The usual assumption is made that each component is approximated by a central mass point and its angular velocity about an axis perpendicular to the orbital plane is identical with the Keplerian angular velocity. Moreover, the \( y \)-coordinate of the point at the intersection of the \( y \)-axis with either equipotential is identified with the fractional radius of each star.

Analysing detached binaries with Main-Sequence components, Kopal (1959) found that the ratio of their Roche constants \( C_1/C_2 \) is nearly equal to the unit independently from their masses and radii. In order to inquire into the possible causes of this fact, the ratio \( C_3/C_2 \) has been calculated for theoretical models of detached binaries with components at various stages of the Main-Sequence band. The calculations have been carried out allowing for a fairly large range of masses and separations.
2. Theoretical Models

Evolutionary models of Iben (1965, 1966a, 1966b, 1966c, 1967a, 1967b), hereinafter referred to as Iben I–VI, have been used (initial chemical composition: $X = 0.71$; $Z = 0.02$).

In order to simplify the construction of isochrone lines, it has been expedient to search for an analytic formula that allows for calculating the stellar radius for every star at any time of its evolution from ZAMS up to the start of the overall contraction phase. By means of the above-cited models (Iben, I–VI), it has been found that the mass-radius relation for ZAMS can be represented as

$$\log R_z = -0.055 + 0.878 \log m - 0.756 (\log m)^2,$$

$$1.0 \, m_\odot \leq m < 2.25 \, m_\odot; \quad (2)$$

$$\log R_z = -0.038 + 0.588 \log m, \quad 2.25 \, m_\odot \leq m \leq 15.0 \, m_\odot;$$

($R$ and $m$ in solar units).

Analogously, on the assumption that stellar radius increases linearly with time (that is an acceptable approximation in the Main-Sequence band), we have

$$\log \dot{R} = -10.364 + 5.026 \log m - 1.330 (\log m)^2,$$

$$1.0 \, m_\odot < m \leq 15.0 \, m_\odot; \quad (3)$$

so that at any stage we obtain

$$R_t = R_z + \dot{R} \tau,$$

where $\tau$ is the time (in years) elapsed from ZAMS.

Fig. 1. Logarithms of the age (in years) for stars at ZAMS ($\tau^{(z)}$) and at the start of the overall contraction phase ($\tau^{(\mathrm{m})}$) as obtained from Iben I–VI.