AN EXPLICIT EXPRESSION OF THE $B_{n,m}^*$-INTEGRALS IN THE
FOURIER ANALYSIS OF ECLIPSING VARIABLES

(Letter to the Editor)

HUSEYIN ALKAN
Dept. of Astronomy, University of Manchester, England*

(Received 13 September, 1978)

1. Introduction

The method of evaluating the photometric effects of the close binary systems with
distorted components has been outlined already in Paper V (Kopal, 1975e); and 'photo-
metric perturbations' $\Psi_{2m}$, arising from the mutual distortion of the components, have
been expressed in explicit form in Paper XVIII (Kopal, 1978).

In earlier investigations 'photometric perturbations' were defined as

$$\Psi_{2m} = \sum_{h=1}^{\Lambda+1} \tilde{C}^{(h)} + \tilde{\Psi}_{2m}^{(h)},$$

where

$$\Psi_{2m}^{(h)} \equiv \int_0^{\theta'} \{ f_{1}^{(h)} + f_{2}^{(h)} + f_{3}^{(h)} \} \, d(\sin 2m \theta);$$

$L_1$ denoting the fractional luminosity of the component undergoing eclipse; $\Lambda$, the
degree of the adopted law of limb-darkening over its apparent disc; and $C^{(h)}$
the functions of the coefficients $U_j (j = 1, 2, \ldots)$ of limb-darkening. The integral of the
right-hand side of Equation (2) consists of three parts; the first two of them are $f_{1}^{(h)}$, and $f_{3}^{(h)}$ (if $h > 1$) were expressed by Kopal (1978, Paper XVIII) in terms of the integrals
of the form

$$A_{n,m}^{a,q} \equiv \int_0^{\theta'} I_0^{a} f_{a}^{e} \, d(\sin 2m \theta),$$

where $m$ is a positive number greater than zero, but not necessarily an integer; and the
last part $f_{3}^{(h)}$, due to the distortion of the shadow cylinder of the eclipsing star, has been
defined by terms of the integrals (cf. Kopal, 1978)

$$B_{n,m}^{a} \equiv \int_0^{\theta'} I_{2}^{a} f_{a}^{e} \, d(\sin 2m \theta),$$

* Permanent address: Atatürk University, Department of Mathematics, Erzurum, Turkey.
for any positive numbers of \( m \) and \( n \) (not necessarily an integer); and the foregoing integral defined by Equation (4), has been evaluated already, for \( q = 0.1 \), in Paper XVIII (Kopal, 1978).

### 2. An Explicit Expression of the \( B_{n,m}^q \)-Integral \((q > 1)\)

In the present work we wish to evaluate the \( B_{n,m}^q \)-integrals, for \( q > 1 \), in terms of the \( A_{n,m}^{p,q} \)'s, which have been expressed in explicit forms in Paper XVIII (Kopal, 1978). The functions of \( I_{n,m}^q \) have also been established as linear combinations of the associated \( \alpha \)-functions of the form \( \alpha_{n+2,2l-4} \), in the Equation (3.15) in the same paper. By means of that equation, \( I_{n,m}^2 \), \( I_{n,m}^3 \), and \( I_{n,m}^4 \)-functions can be obtained as

\[
I_{n,m}^2 = \frac{n}{2} \left( \frac{r_1}{r_2} \right)^{n+1} \mu \alpha_{n-2} + \frac{n + 2}{4} \left( \frac{r_1}{r_2} \right)^{n+2} \frac{r_1}{\delta} \alpha_n
\]

(5)

\[
I_{n,m}^3 = \frac{n}{2} \left( \frac{r_1}{r_2} \right)^{n+1} \mu^2 \alpha_{n-2} + \frac{n + 2}{2} \left( \frac{r_1}{r_2} \right)^{n+2} \left( \frac{r_1}{\delta} \right) \mu \alpha_n + \frac{n + 4}{8} \left( \frac{r_1}{r_2} \right)^{n+3} \left( \frac{r_1}{\delta} \right) \alpha_{n+2}
\]

(6)

where

\[
\mu \equiv \frac{\delta - s}{r_2} = \frac{r_2 - r_1 + \delta^2}{2sr_2}
\]

(8)

On insertion of Equations (5)–(8), in (4), and after a certain amount of algebra, the \( B_{n,m}^q \)-integrals \((q = 2, 3, 4)\) can be expressed in terms of the \( A_{n,m}^{p,q} \)-integrals as

\[
B_{n,m}^2 = \frac{n}{4} \left( \frac{r_1}{r_2} \right)^{n+1} \frac{r_2}{r_1} - \frac{r_2}{r_1} + \frac{1}{r_2} A_{n-2,m}^{0,1} - \frac{n}{4r_2} \left( \frac{r_1}{r_2} \right)^{n+1} A_{n-2,m}^{1,1} + \frac{n + 2}{4} \left( \frac{r_1}{r_2} \right)^{n+2} \frac{r_1}{\delta} A_{n,m}^{0,1}
\]

(9)

\[
B_{n,m}^3 = \frac{n}{4r_2} \left( \frac{r_1}{r_2} \right)^{n+1} \left( \frac{r_2}{r_1} - r_1 + 1 \right)^2 A_{n-2,m}^{0,1} - 2 \left( \frac{r_2}{r_1} - r_1 + 1 \right) A_{n-2,m}^{0,1} + \frac{A_{n-2,m}^{0,1}}{A_{n,m}^{0,1}} \left( \frac{r_2}{r_1} - r_1 + 1 \right) A_{n,m}^{0,1} - A_{n,m}^{0,1} + \frac{n + 2}{4} \left( \frac{r_1}{r_2} \right)^{n+3} \frac{r_1}{\delta} A_{n+2,m}^{0,1}
\]

(10)