ELECTROMAGNETIC RADIATION RECOIL
OF A SYSTEM OF N-CHARGED PARTICLES

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Abstract. We give the radiation of an N-charged particles system associated with the succeeding
terms in the expansion in the inverse powers of the velocity of light of the four vector force on the
material system in a slow motion. We investigate the conditions under which the N-charged particles
system may recoil while emitting electromagnetic radiation. Furthermore, the lowest-order secular
effects in the radiation arise from dipole and quadrupole radiations exactly as it is expected from the
classical theory.

1. Introduction

We consider a slow-motion electromagnetic system of N-charges emitting
electromagnetic waves. Our calculations will be valid for any kind of motion of
N-charges but with the condition that our system remains localized within a
finite volume V. Also we take into account these effects which average to zero
over a long time interval and we denote it with the symbol \( \langle \cdot \rangle \).

The current four-vector satisfies the continuity equation

\[ J^i_{\mu} = 0, \]

where here and henceforth the comma means partial differentiation, and

\[ J^i = (pc, J), \quad \rho = \sum_{n=1}^{n} e_{i} \delta(r - r_{n}), \quad J = \sum_{n} e_{i} v_{i} \delta(r - r_{n}). \]

In this paper the convention will be adopted of letting Latin indices take the
values 0, 1, 2, 3 and the Greek indices take only the values 1, 2 and 3 referring to
the spatial coordinates.

The electromagnetic field tensor is defined by

\[ F_{ik} = \frac{\partial A_i}{\partial x^k} - \frac{\partial A_k}{\partial x^i}, \]

where the meaning of the individual components of the tensor \( F_{ik} \) is easily seen
by substituting the values

\[ A_i = (\varphi, -A) \]

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with the condition
\[ A_i^j = 0 \] (2b)
and \( A_i = g_i^{(0)} A^i, g_{ij}^{(0)} = (1, -1, -1, -1) \). The components of the electric force are \( F_{01}, F_{02}, F_{03} \) and of the magnetic force \( F_{12}, F_{13}, F_{23} \).

2. Flow of Energy and Reaction of the Outgoing Radiation

The four-vector force on the material system is given (cf. Eddington, 1975; Jackson, 1962; Peres, 1962) by
\[ K_i = \frac{1}{c} \int F_{ik} J^k \, dV, \] (3)
where the zero component of \( K_i \) represents the total flow of energy, while the three spatial components represent the net thrust on the material system due to the reaction of the outgoing radiation; that is, we denote the mechanical force vector \( K_i \) by
\[ K_i = \left( \frac{1}{c} K_0, K_i \right), \] (4)
where it should be noted that
\[ K_0 = \int F_{0k} J^k \, dV = \int F_{0\alpha} J^\alpha \, dV; \] (5)
or, simply,
\[ K_0 = \int F_{0\alpha} J^\alpha \, dV. \] (6)

Now, for the spatial components, we have
\[ K_\alpha = \frac{1}{c} \int F_{\alpha k} J^k \, dV = \frac{1}{c} \int F_{\alpha 0} J^0 \, dV + \frac{1}{c} \int F_{\alpha \beta} J^\beta \, dV. \] (7)
Because of Equation (2), Equation (7), becomes
\[ K_\alpha = \frac{1}{c} \int (A_{\alpha, k} - A_{k, \alpha}) J^k \, dV, \quad \alpha = 1, 2, 3. \] (8)
Furthermore, we can put
\[ \int (A_{\alpha, k})_k \, dV = 0 \] (9)
(cf. Peres, 1962; Peres and Rosen, 1960) or by use of Equations (1) and (9) it follows that
\[ \int A_{\alpha, k} J^k \, dV = - \int A_\alpha J^\alpha \, dV; \] (10)