ORIGIN AND EVOLUTION OF THE SOLAR SYSTEM, II

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Abstract. (7) Formation of celestial bodies. The basic concepts of the accretional process are discussed, and the inadequacy of the contractional model is pointed out. A comparison is made between the general pre-planetary state on the one hand and the present state in the asteroidal region on the other. A model for accretion of resonance-captured grains leading to the formation of resonance-captured planets and satellites is suggested.

(8) Spin and accretion. The relation between the accretional process and the spin of planets is analyzed.

(9) Accretion of planets and satellites. It is shown that jet streams are a necessary intermediate stage in the formation of celestial bodies. The time sequence of planet formation is analyzed, and it is shown that the newly accreted bodies have a characteristic internal heat structure; the cases of the Earth and the Moon are considered in detail. A region of high initial temperature is found at 0.4 of the present Earth radius, whereas the culminating temperature of the Moon is near its present surface. An accretional heat wave is found to proceed outwards, and may produce the observed differentiation features.

7. Formation of Celestial Bodies

§ 7.1. Gravitational contraction of a gas cloud

It is generally believed that stars are formed by gravitational contraction of vast interstellar clouds. The condition for contraction is, as regards the order of magnitude,

\[ \kappa M^2/R > nkT, \]  

where \( \kappa \) is the gravitational constant; \( M \), the mass; and \( R \), the radius of the cloud; \( n = M/m_A \) is the number of atoms with average mass \( m_A \); \( k \), Boltzmann's constant; and \( T \), the temperature. If the average atomic weight is \( A = m_A/m_H \), we have

\[ R < 10^{-15} \left( AM/T \right). \]  

As pointed out e.g. by Spitzer (1968), there are serious difficulties in understanding the formation of stars on this model. In particular, a large rotational momentum and magnetic flux oppose the contraction. It is far from certain that the model is appropriate.

However, we shall not discuss here the problem of star formation but the formation of planets and satellites. Laplace made a suggestion, apparently not very seriously intended, that these bodies were formed from gas clouds which contracted gravitationally. This idea has been adopted by a number of subsequent workers, without realization of its inherent inadequacy.

If again for the order of magnitude we put \( A = 10 \) and \( T = 100 \text{K} \) for formation of planets and satellites we find

\[ R < 10^{-16} M. \]
For the biggest planets with $M \approx 10^{30} \text{ g}$ we find $R < 10^{14} \text{ cm}$, indicating that Jupiter and Saturn may have been formed by this mechanism. But already for Uranus and Neptune ($M \approx 10^{29} \text{ g}$) we run into difficulties because gravitational effects do not become important unless the clouds have been caused by some other means to contract to $10^{13} \text{ g}$, which is less than ten percent of the distance between the bodies. Going to the satellite systems or a hypothetical asteroid parent body we see immediately that gravitational contraction is out of the question. For a typical satellite mass, say $10^{23} \text{ g}$, we find $R < 10^7 \text{ cm}$ (which means that the gas cloud should be comparable to the present body).

Hence, we conclude that the gravitational contraction of gas clouds is inadequate as a general model for the formation of the bodies in the Solar System.

As another example which shows how negligible the gravitational attraction is in forming a satellite system, let us consider the inner part of the Saturnian satellite system, certainly one of the most regular examples of a system of secondary bodies. The masses of Mimas and Enceladus are of the order $10^{-7}$ of the mass of Saturn. At a point intermediate between Mimas and Enceladus the gravitation of these bodies is less than $10^{-5}$ of the gravitation of Saturn. Before the formation of the satellites the matter now forming them was spread out over the whole orbit, which makes the ratios still smaller by one or more orders of magnitudes.

The Laplacian approach cannot be saved by assuming that the present satellites once were much larger ('protoplanets' and 'protosatellites' in Kuiper's theory (Kuiper, 1951)). As shown above there are too many orders of magnitude to overcome for such a theory. Moreover, as shown by the isochronism of spins (Sections 5, § 7 and 8, § 3), the idea of very large protoplanets is not acceptable.

§ 7.2. CONDENSATION AND ANGULAR MOMENTUM

The formation of planets and satellites by the gravitational contraction of a gas cloud also meets the same angular momentum difficulty as star formation. If a gas cloud with dimensions $R$ is rotating with the period $T$, its angular momentum is $2\pi R^2 / T$. If it contracts, this quantity is conserved. If the present mass of, say, Jupiter once filled a volume with the linear dimensions $x$ times Jupiter's present radius, its rotational period must have been of the order $T' = T x^2$, where $T$ is the present spin period of Jupiter. If we describe the condensation of Jupiter in a coordinate system which takes part in the orbital motion of Jupiter and hence rotates with the orbital period of Jupiter, the order of magnitude of $T'$ could not be less than the orbital period of Jupiter, which is about $10^4$ times the spin period. Hence, we find $x < 100$, which means that the cloud which condensed to Jupiter must be less than $10^{12} \text{ cm}$. This is only one or two percent of the distance to the point intermediate between Jupiter and Saturn, which should be approximately the separation point between the gas forming Jupiter and the gas forming Saturn. (It is only 10% of the distance to the libration point, which could also be of importance.) Hence, in order to explain the spin of Jupiter if formed by condensation of a gas cloud, one has to invent some braking mechanism. Such a mechanism, however, must have the property of producing the same spin