Abstract. Graviton may, in principle, have a small non-zero mass. In this paper the relevant theory of the massive graviton with six polarisations is developed. The drastic impact of a non-zero mass of the graviton on cosmology is also illustrated.

1. Introduction

Photon and graviton are usually considered as massless fields. Nevertheless, empirically the zero-mass cannot be verified exactly; one can establish only an upper limit (for example, for the mass of photon one has $m_p < 4 \times 10^{-51}$ kg; see Goldhaber and Nieto, 1968). Therefore, one necessarily needs to develop a theory of massive photon as well as graviton. In the case of photon, there is no problem; the massive photon is a standard Proca field with three polarisations (spins are $\pm 1,0$; in this paper we shall set $\hbar = c = 1$). On the other hand, the theory of massive graviton has not yet been developed. For example, Ogievetsky and Polubarinov (1965) proposed several possible theories for massive gravitons.

In this paper we shall solve the problem of the theory of massive graviton. Of course, we do not claim that the mass of graviton is necessarily non-zero; we only point out that, in principle, it may be. In any case, if the mass were non-zero, then the gravity would essentially differ from the Einstein’s gravity on the scales $\geq 1/m$, where $m$ is the mass of graviton. The eventual non-zero mass would essentially change our view of space-time on the scales $\geq 1/m$; the non-zero mass could have a drastic impact on cosmology (we suppose that $1/m$ hardly can be smaller than the present Hubble-radius).

2. The Massive Graviton is not a Standard Massive Spin 2 Field

Einstein’s equations are, in fact, equations of a self-interacting massless spin 2 gauge field; the relevant gauge group is the group of four-translations (cf. Mészáros, 1984b). The equations have the form (Mészáros, 1984b)

$$\square U^\mu_{\nu} - U^{k(\ell,\rho)}_{(k(\ell,\rho)_{\mu})} + U_{\mu_{\nu}} + \eta^\mu_{\nu} U^{km}_{\nu_{km}} - \eta^\mu_{\nu_{\mu_{\nu}}} \square U = \frac{f}{2} t^\mu_{\nu},$$

$$U^\mu_{\nu} = U^\mu_{\nu}, \quad g^\mu_{\nu} = \eta^\mu_{\nu} + f U^\mu_{\nu}, \quad U = U^\mu_{\nu} \eta_{\mu_{\nu}}, \quad f = \sqrt{32 \pi G},$$

$$t^\mu_{\nu_{j}} = 0,$$

where $g^\mu_{\nu}$ is the contravariant metric tensor; $G$, the gravitational constant; $\eta^\mu_{\nu}$,
the Minkowski tensor \((\eta^{ij} = \eta_{ij} = \text{diag}(1, -1, -1, -1))\); \(t^{ij}\), the pseudo-energy-momentum tensor. The \(t^{ij}\)s are, in general, expressible as infinite series of form \(\Sigma_{n=0}^{\infty} f^n(\ldots)\) and the terms contains the potentials \(U^{ij}\) and their first derivatives. For \(t^{ij} = 0\) (1) gives the field equations of free-massless spin 2 field; cf. Pauli and Fierz (1939).

Equations of the standard free massive spin 2 field with \(\pm 2, \pm 1, 0\) polarisations are given by Thirring (1961) as

\[
(\Box + m^2)\; U^{ij} - U^{k(i, j)}_k + U_{, i}^{, j} + \eta^{ij} U^{km}_{, km} - \eta^{ij} (\Box + m^2) U = 0 , \tag{2}
\]
or, alternatively,

\[
(\Box + m^2)U^{ij} = 0 , \quad U^{ij}_{, j} = 0 , \quad U = 0 , \quad m \neq 0 . \tag{3}
\]

Now, it seems, one may immediately introduce the mass term into (1) as follows: the left-hand side of (1) is substituted by the left-hand side of (2). Unfortunately, this way is wrong, because if one had \(t \neq 0\), one would have \(U \neq 0\). Therefore, we have to seek other possibilities.

3. The Massive Spin 2 + 0 Field

In the case of standard massive field, the conditions \(U^{ij}_{, j} = U = 0\) ensure the positive definiteness of total energy and the pure spin 2 field character of field. In what follows we shall show that these conditions are not necessary for the total energy to be positive definite.

Of the field equations

\[
(\Box + m^2)(U^{ij} - \frac{1}{2} \eta^{ij} U) = Q^{ij} , \quad 2U^{ij}_{, j} = U^{, i} , \quad m \neq 0 \tag{4}
\]
fourteen are identical to ten (see the Appendix)

\[
(\Box + m^2)U^{ij} - U^{k(i, j)}_k + U_{, i}^{, j} + \eta^{ij} U^{km}_{, km} - \eta^{ij} \left(\Box + \frac{m^2}{2}\right) U = Q^{ij} , \tag{5}
\]

where the source \(Q^{ij}\) may but need not contain the potentials \(U^{ij}\) themselves. Equations (4) and (5) describe a massive spin 2 + 0 field; beyond the standard polarisations there is a further zero spin yet.

In order to demonstrate this, we consider a free field, for which \(Q^{ij} = 0\). Then one has

\[
U^{ij} = (2\pi)^{-3/2} \int \frac{dk}{\sqrt{2k_0}} (\bar{U}^{ij}(k) e^{-ik_0x^0} + \bar{U}^{ij}(k) e^{ik_0x^0}) ,
\]

\[
k^i = [k^0, \mathbf{k}] , \quad k^i k_i = (k^0)^2 - |\mathbf{k}|^2 = m^2 , \tag{6}
\]