THE NON-AXISYMMETRICAL CONFIGURATION OF A
LARGE-SCALE
SOLAR AND STELLAR MAGNETIC FIELD

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Abstract. The non-axisymmetric and nonlinear solutions of the magnetostatic equations are given in three-dimensional space of spherical coordinates $(r, \theta, \phi)$. These solutions are applied to the large-scale solar magnetic field. Their basic features are similar to a dipole field near the polar regions and the polarity reverses near the equator. These features agree with observations for the large-scale solar magnetic field. The solutions can also be applied to investigating the connection between the structure of the magnetic field and the density distribution of the corona. It is shown that the tops of the closed magnetic field associate with density enhancements.

Similar results may apply to the large-scale configuration of the stellar field.

1. Introduction

It is believed that the solar corona and the large-scale solar magnetic field are three-dimensional, that is, their configurations deviate obviously from the axisymmetric ones. The non-axisymmetric interplanetary magnetic field associates with the non-axisymmetric large-scale solar magnetic field. Observation at low resolution gives the large-scale mean magnetic field of the Sun seen as a star. A magnetic neutral line runs generally north-south in low and middle latitudes and often east-west at high altitudes. Several large-scale regions of alternating polarity near the equator are separated by the neutral line (Wilcox and Howard, 1968; Svalgaard et al., 1974, 1975; Svalgaard and Wilcox, 1978; Levine, 1979).

Many theoretical models have suggested that, based on the approximation of potential field, the line-of-sight component of the photospheric magnetic field is used to determine the large-scale coronal field by the harmonic expansion of the Laplace equation (Altschuler and Newkirk, 1969; Newkirk and Altschuler, 1970; Altschuler et al., 1977; Riebebieter and Neubauer, 1979). On the other hand, if a magnetic dipole rotates obliquely in a vacuum, the polarity-reversal regions of the magnetic field will be formed near the plane perpendicular to the rotating axis. This idea is applied to the three-dimensional structure of the heliospheric magnetic fields (Satio et al., 1978; Kaburaki and Yoshii, 1979). In the solar or stellar atmosphere, the magnetic field is coupled with the plasma. Therefore, the magnetostatic equilibrium should be studied, and the connection between the structure of the magnetic field and the density distribution may be obtained.

Recently, we have analyzed the non-axisymmetric magnetostatic equilibrium for a sunspot-like magnetic field (Hu et al., 1983a, b; Hu, 1983), and nonlinear models are...
also suggested for a small-scale field in the cartesian coordinates (Low, 1983). In this paper, similar approaches are applied to the large-scale solar magnetic field in spherical coordinates. The basic nonlinear theory and the general solution in spherical coordinates are given in the next solution. A special solution is suggested in Section 3. In Section 4, an example is given. The configuration of this magnetic field has polarity reverses near the equator and is similar to a dipole field near the polar region. In the last section, we discuss the coupling relationship between the magnetic field and the plasma.

2. Nonlinear Theory

The magnetostatic equations are

\[ \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla p - \rho \mathbf{g} = 0 , \tag{2.1} \]

\[ \nabla \cdot \mathbf{B} = 0 , \tag{2.2} \]

\[ p = \rho R T ; \tag{2.3} \]

where \( \mathbf{B}, p, \rho, T \) denote the magnetic field, pressure, density, and temperature, respectively; \( R \) is the gaseous constant; and \( \mathbf{g} \) is the gravitational acceleration in the \( \mathbf{e}_r \) direction in the spherical coordinates \((r, \theta, \phi)\).

According to Equation (2.2), the magnetic potential function \( \psi \) may be introduced as

\[ \psi = \frac{1}{r \sin \theta} \left( \frac{1}{r} \frac{\partial \psi}{\partial \theta} - \frac{\partial \psi}{\partial r} , 0 \right) , \tag{2.4} \]

where the azimuthal component of the magnetic field is not included. In this case, the \( \phi \) component of Equation (2.1) reduces to

\[ \frac{\partial}{\partial \phi} (8\pi p + B_\phi^2 + B_r^2) = 0 . \]

Then we have

\[ 8\pi p (r, \theta, \phi) + B_\phi^2 (r, \theta, \phi) + B_r^2 (r, \theta, \phi) = 2a(r, \theta) , \tag{2.5} \]

where \( a \) is an arbitrary function of \( r \) and \( \theta \). Substituting (2.5) into the \( \theta \) component of Equation (2.1) for canceling the pressure \( p \), we obtain

\[ B_r \frac{\partial B_\theta}{\partial r} + B_\theta \frac{\partial B_r}{\partial \theta} = \frac{\partial a(r, \theta)}{\partial \theta} . \tag{2.6} \]

According to definition (2.3), Equation (2.6) becomes

\[ \frac{\partial \psi}{\partial \theta} \frac{\partial}{\partial r} \left( \frac{1}{\sin \theta} \frac{\partial \psi}{\partial r} \right) - \frac{\partial \psi}{\partial r} \frac{\partial}{\partial \theta} \left( \frac{1}{\sin \theta} \frac{\partial \psi}{\partial r} \right) = -r^2 \sin \theta \frac{\partial a(r, \theta)}{\partial \theta} . \tag{2.7} \]