THE STABILITY AND EVOLUTION OF TWO-COMPONENT
ISOTHERMAL CLUSTERS

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Abstract. The stability of two-component isothermal clusters surrounded by a rigid non-conducting spherical wall is examined by linear normal analyses and nonlinear simulations. The examinations are done for four types of models, classified by the differences concerning gravo-thermal stability and Spitzer's condition. Our results show that perturbations in gravo-thermally stable systems disappears with time and the systems tend to isothermal ones with equipartition, as is expected. On the other hand, in the gravo-thermally unstable systems, the presence of small amount of massive component which has higher central density accelerates the gravo-thermal collapse by heat flowing from the massive component to the less massive component and being transported outward efficiently. This effect of the interaction between two components on gravo-thermal collapse is shown clearly in the forms of the respective eigenfunctions.

1. Introduction

The major process which makes spherical stellar clusters evolve is the spatial heat flow due to gravitational encounters among stars. This gives rise to gravo-thermal instability when the density concentration to the center exceeds a certain critical value, as first shown by Antonov (1962) and demonstrated intuitively by Lynden-Bell and Wood (1968).

In multi-component systems, however, there is another situation which facilitates the gravo-thermal collapse. This is the heat transport among subsystems of different mass components. It is known (Spitzer, 1969) that the global thermodynamical equilibrium among different mass components cannot be realized in self-gravitating systems with no outer boundary, if a certain condition (Spitzer's condition) is not satisfied. In other words, unless the condition is satisfied, heat flow among different subsystems is inevitable. This promotes the contraction of subsystems of massive stars, which gives rise to gravo-thermal instability of the whole system and accelerates the evolution due to the instability.

In this sense, understanding of stability and evolution of two-component stellar clusters is of importance to know the evolution of real clusters. There are many studies on this problem (e.g., Saito and Yoshizawa, 1976; Yoshizawa et al., 1978; Inagaki and Wiyanto, 1984). The purpose of this paper is to make a contribution to this problem by comparing in detail stability and evolution of the following four qualitatively different two-component stellar models.

From the point of view of evolution of systems, the major distinction of stellar models is whether they are gravo-thermally unstable or not. The second minor distinction is

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whether the systems satisfy Spitzer's condition or not. By combining these two distinctions, we can construct four different types of stellar models. In this paper, stability and nonlinear time evolution of typical models of these four types of clusters are examined.

In Section 2, we explain in detail four models which we choose to study their stability and evolution. In Section 3 the results of linear stability analyses of these four models are given. Their nonlinear evolutions are examined in Section 4, briefly by solving the two-dimensional Fokker–Planck equation, and more extensively by solving the one-dimensional one. Section 5 is devoted to discussion.

2. Models

Two-component isothermal stellar clusters considered here are enclosed by a rigid spherical wall having no heat exchange with the outside. The radius of the wall is \( r_e \). The individual mass of less massive stars is \( m_1 \) and that of massive stars is \( m_2 \). The total mass of less massive and massive stars are, respectively, \( M_1 \) and \( M_2 \).

Throughout this paper, the ratio \( m_2/m_1 \) is taken as \( m_2/m_1 = 4 \). Then for a given \( M_2/M_1 \), there is a series of equilibrium models. It is described by a curve on the dimensionless \((1 + \lambda)^{-1} - z_e \) plane. Here \( \lambda \) is the ratio of central density of the massive to the less massive components, i.e., \( \lambda = \rho_2(0)/\rho_1(0) \), and \( z_e = (4 \pi G \rho_1(0) m_1 k T)^{1/2} r_e \), where \( T \) is the temperature defined by \( 3kT = m_1 \langle v_1^2 \rangle = m_2 \langle v_2^2 \rangle \), \( \langle v_1^2 \rangle \) and \( \langle v_2^2 \rangle \) being, respectively, r.m.s. velocities of less massive and massive stars. For various values of \( M_2/M_1 \), the curves on the \((1 + \lambda)^{-1} - z_e \) plane are shown in Figure 1. The corresponding figure in the case of \( m_2/m_1 = 10 \) is given by Yoshizawa et al. (1978).

The model of marginal stability against gravo-thermal instability is obtained by the linear series method (Lynden-Bell and Wood, 1968). The method can be extended to the case of two-component clusters (Yoshizawa et al., 1978). In the present case of rigid, perfectly reflecting wall, the method tells us that the marginal model is the one for which the total energy \( E \) of the cluster is extremum (a turning point) on the equilibrium series with constant \( M_1, M_2, \) and \( r_e \). The positions of marginal models obtained by this method are shown in Figure 1 by marks of square. On this figure, the upper left part of the curve connecting the squares is the region of models which are gravo-thermally stable. On the other hand, the lower right part is the region of unstable models.

It is noted that in the next section the stability of models is examined by the linear stability analyses. The positions of the marginally stable models shown in Figure 1 are found to coincide well with those derived independently by the linear stability analyses.

Here we adopt two different values of \( M_2/M_1 \). One is \( M_2/M_1 = \frac{1}{4} \) and the other is \( M_2/M_1 = \frac{1}{99} \). The clusters with \( M_2/M_1 = \frac{1}{4} \) do not satisfy the Spitzer condition, \( M_2/M_1 < 0.16 (m_2/m_1)^{-3/2} \), while those with \( M_2/M_1 = \frac{1}{99} \) do. This implies that the clusters with \( M_2/M_1 = \frac{1}{4} \) cannot become a thermodynamically equilibrium system by their own self-gravity: the equilibrium has been realized only by the presence of the surface pressure. In other words, the less massive component (this is the major component of the system) is not self-gravitating in the outer region of the system. On