THE TRANSFER OF POLARIZED RADIATION IN SPECTRAL LINES: FORMALISM AND SOLUTIONS IN SIMPLE CASES

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Abstract. A general treatment of the transfer of polarized radiation in spectral lines assuming a Rayleigh phase function and a general law of frequency redistribution is derived. It is shown how nine families of coupled integral equations for the moments of the radiation field arise which are necessary to fully describe the state of polarization of the emergent radiation from a plane-parallel, semi-infinite atmosphere. The special case of angle independent redistribution functions is derived from the general formalism, and it is shown how the nine families of integral equations reduce to the six linearly independent integral equations derived by Collins (1972). To serve as a test of the formulation, solutions for isothermal atmospheres are given.

1. Introduction

It has been shown by a number of investigators (Collins and Harrington, 1966; Collins, 1968a, b; Hardop and Strittmatter, 1968; Collins, 1970a; Slettebak et al., 1980) that a great deal of information can be obtained about the atmosphere of rotating stars from the shapes and depths of the absorption lines. In particular, different parts of the star contribute in a non-uniform manner to the strength and shape of a given spectral line due to the temperature and pressure variation across the surface of the star. Thus, to an external observer, the general appearance of a line will depend upon the orientation of the system. However, with only the emergent flux, one cannot untangle the speed of rotation and the angle of inclination of the system since there is only the one piece of information. This can be rectified, to some extent, given more information about the emergent radiation. Harrington and Collins (1968) have shown that one can expect measurable degrees of polarization of the continuum polarization, the size of which will depend upon the orientation of the system.

The situation will differ somewhat for spectral lines. The gradient of the source function in lines can be quite large. Thus, the radiation field is locally more anisotropic leading to possibly large degrees of polarization. Further, since the redistribution in the line changes from complete redistribution to coherent scattering (depending upon the actual nature of the scattering mechanism), the degree of polarization will vary across the line. Averaging over the entire star will dilute the degree of polarization somewhat; but, if we recall that the star's surface is not at a uniform temperature and pressure, there should be a net polarization at each frequency within the line.

The ultimate purpose of this paper is to investigate the wavelength dependence of the emergent Stokes parameter across a spectral line for rotating stellar atmospheres. The
aim of this paper is to develop a rather general NLTE formalism for solving for the emergent Stokes parameters across a spectral line from a plane-parallel, semi-infinite atmosphere.

The theoretical development of the transport of polarized radiation in spectral lines has been slow in developing. This is in part due to the coupling of the Stokes intensity components \((I_0, I_r, U, V)\) which forces the simultaneous solution of the coupled transport equations for each Stokes component. For problems in which the radiation field is not azimuthally symmetric and where there is a coupling in frequency space, the solution of the transport equations becomes quite difficult numerically.

In developing the theoretical basis for radiation transport, there are two basic approaches which may be adopted. One could opt to solve the integro-differential equation for the specific intensity vector. To this end there are several different variations (see, for example, Stenflo and Stenholm, 1976; Stenflo, 1978; Dumont et al., 1977; Auer et al., 1977; Landi Degl'Innocenti, 1978; Brown and McLean, 1977; Brown et al., 1978; and McLean and Brown, 1978). Alternatively, one could use the integral equation for the source function together with the classical solution of the equation of transfer of the radiation field from which one can determine the emergent radiation field (Harrington, 1970; Collins, 1972; Collins and Buerger, 1974; Rees, 1978). The first method involves the simultaneous solution of the intensity vector at all \(x, \theta, \phi, \tau\) in the four dimensional grid of points, and these are the only points where one has the complete solution. With this data, one can compute, by numerical integration, the observables. With the moment method, one is directly solving for the moments of the radiation field. From these, one immediately has the source function at the chosen \((x, \theta, \phi, \tau)\) from which one can compute the emergent intensity at any \((x, \theta, \phi, \tau)\) using the classical solution of the transfer equation. The physical moments, such as the flux or the mean intensity, are simply linear combinations of the general moments.

For cases where azimuthal symmetry cannot be assumed and where there is coupling of the radiation field in frequency space due to photon redistribution upon scattering, the moment equation is preferable. This is of course due to the nature of the moments which by definition are angle independent. One is thus effectively separating the angle dependences from the depth and frequency dependences.

The moment equation method to be derived in this paper is a generalization of the method discussed by Collins (1970, 1972) and Collins and Buerger (1974). The stellar atmosphere program ATLAS 6 has been modified in such a way that the radiative transfer is done using the method described by Collins (1972). Studies involving the continuum polarization of rotating stellar atmospheres (Harrington and Collins, 1968; Collins, 1970), and studies of polarization arising from close binary systems (Buerger and Collins, 1970) show that this basic technique is both powerful and versatile.

McKenna (1978) has investigated the moment equation method for the case of an isotropic phase function and an angle dependent redistribution function. For complete redistribution with Doppler and Voigt functions as well as \(R_I\) and \(R_{II}\), the moment method was found to be accurate and very stable in its numerical behaviour.

Let us examine the situation a little more closely. The equation of transfer for