PROPAGATION OF WEAK DISCONTINUITIES IN A VIBRATIONALLY-EXCITED GAS FLOW

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Abstract. Propagation of weak discontinuities headed by wavefronts of arbitrary shape in three dimensions are studied in vibrationally relaxing gas flow. The transport equations representing the rate of change of discontinuities in the normal derivatives of the flow variables are obtained, and it is found that the nonlinearity in the governing equations does not contribute anything to the vibrationally relaxing gas. An explicit criterion for the growth and decay of weak discontinuities along bi-characteristic curves in the characteristic manifold of the governing differential equations is given. A special case of interest is also discussed.

1. Introduction

Considerable attention has been given to the development of non-equilibrium gas flow theories which have applications in the fields of entry-physics, combustion, and reaction. The non-equilibrium effects can occur in any of the molecular processes, translation, rotation, vibration, chemical transformations, etc. The mechanism of vibrational relaxation can be understood by assuming a system of oscillators in equilibrium state with the heat bath. Suppose that the equilibrium distribution is disturbed by any external agency. The oscillators continue to change from one energy state to another upon collisions with the heat bath. When the source of disturbance is removed, the system of oscillators approaches its equilibrium value. This change requires a length of time. Hence, this rate process of non-equilibrium is called vibrational relaxation (cf. Vincenti, 1965).

One of the most interesting part of the theory of propagation of weak discontinuities is that it is a subclass of nonlinear waves which admit analytical solutions. The distortion of wave propagating in various relaxing gases due to non-equilibrium effects has been a subject of a number of researchers during the last two decades, Chu (1958), Moore and Gibson (1960), Wegener and Chu (1965), Sussman and Baroh (1967), and Blythe (1969). Theoretical investigations on the growth of acceleration waves with relaxation effects were carried out by Becker (1970), Bowen and Chen (1972), Becker and Schmitt (1968), Rarity (1967), and Ram (1979). But most of these investigators made a simplified assumption of one-dimensional flow, and the wave was assumed to propagate normally to itself at a constant gas at rest. They could not spell out the behaviour of acceleration waves of arbitrary shape propagating arbitrarily into non-uniform, inhomogeneous, and non-equilibrium state of fluid motion.

The main aim of the present communication is to study the behaviour of weak discontinuities in a vibrationally relaxing 3-D gas flows along bi-characteristics and to
study the nature of the weak wave in a uniform medium. Prasad (1975) and Elcrat (1977) also studied the propagation of weak discontinuities along bi-characteristics in inviscid gas flows without accounting for relaxation effects. In this paper, the analysis is based entirely upon the theory of singular surfaces which in comparison to the theory of characteristics, quickly leads to result of general significance. We consider a moving singular surface $\Sigma(t)$ of a weak discontinuity called a 'sonic wave' across which the flow parameters are continuous themselves but they undergo jumps in their first- and higher-order derivatives. The boundary conditions are

$$[Z] = 0, \quad [Z, i] \neq 0, \quad \left[ \frac{\partial Z}{\partial t} \right] \neq 0,$$

where $Z$ stands for any of the flow quantities and $[Z] = Z_0 - Z_1$ in which $Z_1$ denotes evaluation of $Z$ on the wavefront as approached from the downstream side and $Z_0$ denotes evaluation of $Z$ on the wavefront as approached from the upstream side. A comma followed by an index denotes partial differentiation. The geometrical and kinematical compatibility conditions of first order due to Thomas for sonic discontinuities are those of Thomas (1958) – i.e.,

$$[Z, i] = Bn_i, \quad \left[ \frac{\partial Z}{\partial t} \right] = -BG,$$

where $B$ is a scalar function defined over $\Sigma(t)$, i.e., $B = [Z, i]n_i$; $G$ is the speed of propagation of the surface $\Sigma(t)$ into a medium. $n_i$ are the components of the unit normal to the wave surface in the direction of propagation.

2. Law of Propagation

In formulating the equations of motion, we adopt the heat sink analogy of Johannesen (1961) which exploits the exact correspondence between the flow of a gas of variable specific heats with relaxation and the flow of a gas at constant specific heats, Johannesen’s alpha gas, to which heat is added or from which heat is extracted at the rate at which energy is released or absorbed by the vibrational mode of the real gas. Thus the entropy $S$ of the gas is related to $p$ and $\rho$ by the relation

$$\left( \frac{p}{p_0} \right) = \left( \frac{\rho}{\rho_0} \right)^\gamma \exp \left( (S - S_0)/C_v \right),$$

where $C_v$ is the specific heat per unit volume and $\gamma$ is the heat exponent of the gas. The suffix ‘0’ denotes values of the reference state.

Now we consider the phenomena of vibrational relaxation due to collisions in the flow field. Specifically, we consider a system of harmonic oscillators with vibrational energy $\sigma(x', t)$. This system of harmonic oscillators is assumed to be capable of collision, interchange of energy with the translational degree of freedom of a heat bath. Such a heat bath can be provided either by a large excess of inert gas containing a small concentration of the diatomic species or by the translational and rotational degrees of