AN UPPER LIMIT TO THE DENSITY INHOMOGENEITY IN THE UNIVERSE

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Abstract. An upper limit to the amplitude of the overall density fluctuation has been found by means of the gravitational lensing effect of the density inhomogeneity on the luminosities of quasars with larger redshifts. The observed differences of luminosities of quasars located at different directions are partially given by the lensing effect, therefore, a useful upper limit to the inhomogeneity can be derived if the luminosity distribution of quasars is uniform enough. We obtain that, in the case of the density parameter of the Universe $\Omega_2 = 1$, the overall matter should be less clustered than the luminous matter by a factor of at least 3. This result may not favour the biased dark matter scenario for the formation of large-scale structure in the Universe.

1. Introduction

One of the most important problems in observational cosmology is to determine the density inhomogeneity in the Universe. Cosmic matter distributes homogeneously only on a very large scale, while on the scale of less than about a few 100 Mpc matter exists in the form of galaxies, clusters of galaxies, and superclusters. The distribution of such visible objects shows considerable inhomogeneity. All the amplitudes in the length scales of galaxies, clusters of galaxies and superclusters are larger than or about the same as unity: namely, the inhomogeneity has already evolved into nonlinear stage of clustering.

However, in a dark-matter dominant universe, the total density inhomogeneity would be dominated by the density distribution of dark matter. Therefore, information about the distribution of dark matter is necessary in studying the large-scale structure of the Universe. Obviously, it is more difficult to determine the length scale and the amplitude of the inhomogeneity of the dark matter. Nevertheless, some information of such inhomogeneity has been obtained from the gravitational effects of the dark matter. For instance, from the distribution of galaxies in the local superclusters and the peculiar velocity of the local group in the direction of the Virgo cluster one concludes that the density distribution of dark matter must be more uniform than that of galaxies. The same result was obtained by the statistical approach for the entire galactic population considered as a field of fluctuating density in random motion with respect to the Hubble flow (Fang et al., 1982).

In this paper we present an upper limit to the amplitude of the inhomogeneity of total (visible plus dark) mass in the Universe, by means of the gravitational lensing effect of the density inhomogeneity on quasar luminosities.

The stochastic perturbation in gravitational field due to density inhomogeneity will lead to luminosity fluctuation for distant sources. Therefore, the amplitude of the...
inhomogeneity can be deduced from the observed differences in luminosities between sources which have originally the same luminosity. The theory of this method has been discussed by one of us a few years ago (Fang and Sato, 1982; Fang, 1983). The difficulty in using this method is the lack of available sources with the same luminosity.

Recently, the spatial distributions of quasars from many catalogues and surveys have been analysed (Osmer, 1981; Webster, 1982; Chu and Zhu, 1983; Shanks et al., 1983; Fang et al., 1985). All the results show that the distribution of quasars is quite uniform or of very weak clustering, especially for the quasars with larger redshifts (Fang et al., 1985). This shows that quasars may be used as the required sources. Of course, the intrinsic luminosities of quasars are different. However, the lensing effect is one of the reasons for the observed differences in luminosities of quasars, so an upper limit to the amplitude of inhomogeneity can be derived, supposing that all the luminosity inhomogeneity of quasars comes from the lensing effect. We will show that a useful upper limit to the density inhomogeneity can already be found in this way.

2. Luminosity Fluctuation Due to Density Inhomogeneity

The propagation of radiation will be affected by the metric perturbation of density inhomogeneity, so the flux fluctuation will be given by the stochastic small gravitational deflection. It is also called the multiple lensing effect, namely, a beam of light affected simultaneously by several lenses. This effect is observable as a luminosity difference between sources with angular distance of the order of

\[ \vartheta \sim \lambda/d, \]

where \( d \) is the distance of the sources and \( \lambda \) is the scale-length of the inhomogeneity. The light signals from the sources have experienced a statistically independent stochastic field if their source directions are different. The luminosity fluctuation comes from these different histories of the propagations (Young, 1981; Fang and Sato, 1982).

The mean-square-fluctuation of luminosity is given by

\[ \left( \frac{\Delta I}{I} \right)^2 = \frac{2}{3} B d^3, \]

where

\[ B = 2h^2 \int_0^\infty \left[ \frac{1}{r^2} \frac{d^2}{dr^2} M(r) - \frac{1}{r^3} \frac{d}{dr} M(r) \right] dr. \]

The quantities \( h^2 \) and \( M(r) \) can be expressed by the density perturbation spectrum \( \delta_k \) (\( k \) being the wave number of the perturbation) as

\[ h^2 = \left( \frac{6\pi G \rho}{c^2} \right)^2 k^{-4} \delta^2, \]