ON NUMERICAL EVALUATION OF THE H-FUNCTIONS OF TRANSPORT PROBLEMS BY KERNEL APPROXIMATION FOR THE ALBEDO \(0 < \omega \leq 1\)

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Abstract. Das Gupta represented the H-functions of transport problems for the albedo \(0 < \omega \leq 1\) in the form

\[ H(z) = R(z) - S(z) \]

(see Das Gupta, 1977) where \(R(z)\) is a rational function of \(z\) and \(S(z)\) is regular on \([-1, 0]\). In this paper we have represented \(S(z)\) through a Fredholm integral equation of the second kind with a symmetric real kernel \(L(y, z)\) as

\[ S(z) = f(z) - \int_0^1 L(y, z)S(y)\,dy. \]

The problem is then solved as an eigenvalue problem. The kernel is converted into a degenerate kernel through finite Taylor’s expansion and the integral equation for \(S(z)\) takes the form:

\[ S(z) = f(z) - \sum_{j=1}^{\infty} \int_0^1 F(y)S(y)\,dy \]

(which is solved by the usual procedure) where \(\lambda_j\)’s are the discrete eigenvalues and \(F_j\)’s the corresponding eigenfunctions of the real symmetric kernel \(L(y, z)\).

Das Gupta obtained the Chandrasekhar’s H-function \(H(z)\) in the form (see Das Gupta, 1977, Equations (61), (62), (65), (98)) which separates the pole from the branch points of \(H(z)\) as

\[ H(z) = R(z) - S(z), \]

where

\[ R(z) = \begin{cases} \frac{(A_0 + H_0z)(K + z)}{K + z} & \text{when } 0 \leq \omega < 1, \\ h_1z + h_0 & \text{when } \omega = 1, \end{cases} \]

(2a)

(2b)

\[ S(z) = \sum_{s=1}^{n} \int_{E_s} \frac{P_s(x)}{x + z} \,dx \]

\[ = \int_0^{\lambda_s} \frac{P(x)\,dx}{x + z}, \quad E_s = [\lambda_s, \lambda_{s+1}], \]

(3)

\[ P(x) = P_s(x) \quad \text{when } x \in E_s, \quad s = 1, 2, \ldots, n, \]

(4)

\[ P_s(x) = \phi_s(x)/H(x), \]

(5)

defined on \(E_s\),
\[ \phi_s(x) = \left( \frac{1}{\pi} \right) U_s(x)/[T_s^2(x) + U_s^2(x)] , \] (6)

\[ T_s(x) = F_1(x) - \sum_{p=1}^{s} Y_p(x) x \ln \left[ (\lambda_p + x)/(\lambda_p - x) \right] - \sum_{p=s+1}^{n} Y_p(x) x \ln \left[ (x + \lambda_p)/(x - \lambda_p) \right] , \] (7)

\[ U_s(x) = \pi \sum_{p=1}^{s} x Y_p(x) , \quad U_1(x) = \pi x Y_1(x) , \] (8)

\[ F_1(x) = 1 - \sum_{p=1}^{\lambda_1} 2x^2 \int_{0}^{\lambda_p} [Y_p(t) - Y_p(x)] \, dt/(x^2 - t^2) , \] (9)

\[ A_0 = (1 + p_{-1}) K , \] (10)

\[ p_{-1} = \int_{0}^{\lambda_1} \left[ P(x)/x \right] \, dx , \] (11)

\[ H_0 = (1 - \omega)^{1/2} , \quad h_1 = (1/2)^{1/2} , \quad h_0 = h_1 \left[ (\gamma_2/\gamma_1) + q_0 \right] , \] (12)

\[ \gamma_r = \sum_{s=1}^{n} \int_{0}^{\lambda_s} x^r Y_s(x) \, dx \] (12a)

and (see Das Gupta, 1974, Equation (28b))

\[ q_0 = \sum_{p=1}^{n} \int_{E_p} Q_p(x) \, dx , \] (13)

\[ Q_p(x) = \left( \frac{1}{\pi} \right) \tan^{-1} \left[ \frac{v_p(x) - u_p(x)}{1 + v_p(x) u_p(x)} \right] , \] (14)

\[ v_r(x) = 2U_r(x)/L_r(x) , \quad u_r(x) = U_r(x)/T_r(x) , \] (15)

\[ L_r(x) = F_2(x) - \sum_{p=1}^{r} Y_p(x) \ln \left[ (\lambda_p - x)/x \right] - \sum_{p=s+1}^{n} Y_p(x) x \ln \left[ (x - \lambda_p)/x \right] , \] (16)

\[ F_2(x) = 1 - \sum_{s=1}^{n} 2x \int_{0}^{\lambda_s} [Y_s(t) - Y_s(x)] \, dt/(x - t) . \] (16a)