EFFECTS OF SOLAR RADIATION ON THE ORBITS OF SMALL PARTICLES

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Abstract. Revised equations of motion are formulated on more general assumptions than hitherto making allowance for some reflection of sunlight by a dust-particle, and from these the secular rates of change of the orbital elements of the particle are obtained. The equation for the eccentricity yields numerical results for the time taken for given changes in this element to occur. Other elements turn out to be expressible in terms of the eccentricity and thence are effectively also known in terms of the time. More general forms of earlier results are found, and some new mathematical results in the theory of the process are derived. The time of infall to the Sun associated with almost circular initial motion of a particle is calculated, and also the time from an orbit of initially high eccentricity. In this latter case, infall takes place much more rapidly than from a circular orbit of radius comparable with the average distance in the eccentric orbit. The effect on a particle of a long-period comet during a single return is negligible compared with the change in its binding-energy to the Sun that will in general result from planetary action. The possible history of a dust-particle from original capture by the Sun to final infall to the solar surface is briefly considered.

1. Introduction

Investigation of the dynamical effects on the orbit of a small particle in the solar system resulting from its interaction with solar radiation seems first to have been undertaken by Poynting (1903) at a time when only classical methods of radiation-theory were available. The fixed aether had not lost its hold, but radiation-pressure had been recognized, and the phenomenon of aberration was also introduced in attempts to arrive at the appropriate forces, which were put together more or less piecemeal by various considerations. In addition to the direct repulsive action of the pressure of radiation on the particle, which takes the form of reducing the strength of solar gravitation \( \mu_0 = GM \) (where \( G \) = constant of gravitation, and \( M \) = mass of the Sun), other forces arise that depend on the orbital velocity of the particle. The general form of the equations of motion emerges on almost any basis of their derivation, but Plummer (1905, 1906) found numerical coefficients in the velocity-terms different from those obtained by Poynting, while subsequently Larmor (1913, 1917) claimed still different values. The problem remained in this unsettled state for two decades until it was taken up afresh by Robertson (1937) by means of the special theory of relativity, which had meanwhile become well established. His main objective was to give a rigorous derivation of the equations of motion correct to the first order in the ratio of the velocity of the particle to the velocity of light. Interestingly enough, the equations...
so obtained demonstrated that none of the earlier forms had in fact been correct in their numerical factors associated with the velocity-terms.

The present paper has the object first of considering possible modifications of the Robertson-equations for the case when the particles have non-vanishing albedo and reflect some of the solar radiation directly into the sunward hemisphere. The more general equations expressing this situation can nevertheless be solved by standard perturbation methods to give the long-term changes in orbital elements and the time-intervals over which large changes in the elements would occur. It is found that infall to the Sun from a highly eccentric orbit proceeds much more rapidly than from an almost circular orbit of comparable size.

2. The Equations of Motion

The properties assumed for the particle by these earlier authors and by Robertson were that the particle was spherical in shape, that it absorbed all radiation intercepted by it, and that it re-emitted this radiation isotropically in the proper-frame of the particle. On this basis, the final equation of motion derived by Robertson (1937, Equation 3.3) was equivalent to

\[ m \frac{dv}{dt} = f \left( 1 - \frac{v \cdot \mathbf{n}}{c} \right) \mathbf{n} - f \frac{v}{c}, \quad (1) \]

where \( m \) = mass of particle = \( \frac{4}{3} \pi \sigma s^3 \), \( \sigma \) = density and \( s \) = radius,
\( v \) = orbital velocity of particle,
\( c \) = velocity of light,
\( \mathbf{n} \) = unit vector in direction of increasing \( r \) (= distance from the Sun),
\( f = m c^2 / r^2 \),
\( x = A S b^2 / mc^2 = 2.54 \times 10^{11} / \sigma s \) in c.g.s. units, \( b = 1 \) AU = 1.496 \times 10^{13} cm,
\( A \) = effective cross-section of particle = \( \pi s^2 \),
\( S \) = solar constant (at distance \( b = 1 \) AU) and = 1.36 \times 10^6 erg s\(^{-1}\) cm\(^{-2}\).

The numerical values adopted for \( c, b, \) and \( S \) are as given by Allen (1973).

It is readily shown from (1) that the path of the particle always remains in a fixed plane which is that defined by the radius vector \( \mathbf{r} \) from the Sun and the velocity-vector \( \mathbf{v} = \mathbf{v} \). If the particle is referred to the Sun by polar coordinates \( (r, \theta) \) in this plane, then (1) is equivalent to

\[ \ddot{r} - r \dot{\theta}^2 = -\frac{\mu}{r^2} - \frac{2 x \dot{r}}{r^2}, \quad (2) \]

\[ r \ddot{\theta} + 2 r \dot{\theta} = \frac{\alpha x \dot{r}}{r^2}, \quad (3) \]

where in (2)
\[ \mu = GM - \alpha c = \mu_0 - \alpha c. \quad (4) \]

These are the definitive equations arrived at by Robertson (1937, p. 431).