CRITICAL ANGULAR VELOCITY OF RIGIDLY ROTATING WHITE DWARFS

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Abstract. The equilibrium configurations of rigidly rotating white dwarfs are calculated numerically as an application of the finite difference – finite expansion method pioneered by Stoeckly. The latest version of the Harrison–Wheeler equation of state is used, together with the post-Newtonian equations of structure. No other approximation is made. The resulting critical values for the angular velocity agree in order of magnitude with a ‘crude’ approximation to these values by Hartle and Thorne, but fractional differences in mean radius and in mass and eccentricities are very different.

1. Introduction

Equilibrium configurations of cold white dwarfs have been calculated by a number of authors in the case when these stars are not rotating (see, for example, Harrison et al., 1965). The effects of rigid rotation on the structure have been investigated by Hartle and Thorne (1968). They consider rotation slow enough that the general-relativistic equations of structure can be expanded in powers of the angular velocity and terms of higher order than second neglected. Such a perturbation approach is expected to fail before the critical angular velocity is reached, when white dwarfs develop a cusp at the equator.

In this paper the exact structure equations are solved numerically using Stoeckly’s finite difference-finite expansion method. The structure of white dwarfs is known to be Newtonian (see Weinberg, 1972 and Misner et al., 1973) except near the Chandrasekhar limit, so that the post-Newtonian formalism is used for obtaining the structure equations. This approach results in values of critical angular velocity that are smaller (by about a factor of 2) than the estimates obtained by Hartle and Thorne and in fractional differences in mean radius and mass, and eccentricities that are considerably different.

2. Equations of Structure

It has been shown by Krefetz (1967) that the post-Newtonian equations of structure determining an equilibrium configuration of a self-gravitating and rigidly rotating (say about the z-axis) matter are

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\[
\left(1 - \frac{1}{c^2} (\Pi + p/\rho)\right) \nabla p = \rho \nabla \left\{ U + \frac{1}{2} \Omega^2 \omega^2 + \right.
\]
\[
+ \frac{1}{c^2} \left[ \frac{1}{2} \left( \Omega^2 \omega^2 \right)^2 + 2 \Omega^2 \omega^2 U + 2 \Phi - 4 \Omega^2 \omega^2 D \right] \},
\]
\[
\nabla^2 U = -4\pi G \rho,
\]
\[
\nabla^2 D + \frac{2}{\omega} \frac{\partial D}{\partial \omega} = -4\pi G \rho,
\]
and
\[
\nabla^2 \phi = -4\pi G \rho (\Omega^2 \omega^2 + U + \frac{1}{2} \Pi + \frac{3}{2} p/\rho),
\]
where \( \rho \) is the (rest) mass density, \( p \) the pressure, \( \rho \Pi \) the internal energy density, \( \Omega \) the angular velocity, and \( \omega \) the distance from the axis of rotation. \( U, D, \) and \( \Phi \) are gravitational potentials.

In addition to Equations (1), an equation of state specifies the pressure and the internal energy density as functions of the density. For white dwarfs this relation will be one of the equations of state for cold degenerate matter. In this paper the Harrison-Wheeler equation of state (see Harrison et al., 1965) is used in a tabular form adapted for nonrelativistic computations. Table I presents this equation of state. The entries in the table are given by the following equations
\[
\rho = (E^* - e)/0.742 \times 10^{-28}, \quad p = p^*/0.742 \times 10^{-28}
\]
\[
\Pi = c^2 e^*/(E^* - e^*),
\]
where the values for starred quantities are taken from Hartle and Thorne (1968). Interpolation between points in the table is made assuming a constant logarithmic derivative between, i.e. for \( \rho_j < \rho < \rho_{j+1} \)
\[
\frac{\log p - \log p_j}{\log \rho - \log \rho_j} = a
\]
is taken as constant and equal to
\[
a = \frac{\log p_{j+1} - \log p_j}{\log \rho_{j+1} - \log \rho_j}.
\]
Then
\[
p = p_j \left( \frac{\rho}{\rho_j} \right)^a;
\]
and, because
\[
\frac{d \log p}{d \log \rho} = \frac{\rho}{p} \frac{dp}{d\rho},
\]
\[
\frac{dp}{d\rho} = a \frac{p}{\rho}.
\]
(3)
This derivative occurs frequently in the computations.