THE FORMATION OF POLAR AND EQUATORIAL
CONDENSATIONS OF PLASMA IN THE PROXIMITY OF A
CHANGING MAGNETIC DIPOLE*

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Abstract. The motion of a plasma in a time-dependent dipole magnetic field is considered. It is shown that the increase of the magnetic moment of the dipole (for example, as the result of the explosion of a magnetic star) leads to a concentration of a plasma in the polar regions. Likewise, a decrease of the magnetic moment (contraction of a star) would lead to the concentration of surrounding plasma in the equatorial plane. This process may be of importance in astrophysics and, particularly, in the dynamics of nebulae and non-stationary star envelopes.

A class of astrophysical objects of true nonspherical form are known to exist, for which polar condensations or equatorial rings are characteristically present. For example, some flaring stars (Baade, 1940; Boyarchuk and Mustel, 1968; Vorontsov-Velyaminov, 1948) and planetary nebulae (Vorontsov-Velyaminov, 1948; Hromov and Kohoutek, 1968) belong to this class. It is difficult to explain such formations only by the action of the force of gravitation and nonuniform rotation. Apparently, the magnetic field plays a definite role in the origin of these forms (Mustel, 1956). Below, the motion of rarefied plasma surrounding a central body with a variable magnetic moment (i.e., a nonstationary star) is examined. It is shown that the growth of the magnetic moment of the central body (explosion and expansion of a star) is accompanied by the appearance of polar condensations. On the other hand, with a decrease of magnetic moment (contraction of a star) the gas surrounding the star becomes concentrated in the equatorial plane, i.e. an equatorial condensation arises. The calculations are given within the framework of the magnetohydrodynamics of an ideal compressible fluid in the approximation of small slow changes of the magnetic field.

In what follows, we shall consider some body (e.g., a star) generating an axially symmetric poloidal magnetic field whose field lines lie in meridian plane. Because of the axial symmetry, the magnetic field strength may be expressed by means of the vector potential \( \mathbf{A} \) which in spherical coordinates has only a \( \phi \)-component \( A_\phi \), or by means of a 'current function'

\[
\phi (r, \theta, t) = A_\phi (r, \theta, t) r \sin \theta .
\]

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Then
\[ H_r = \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta A_{\theta} \right) = \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta}, \]
\[ H_{\theta} = \frac{1}{r} \frac{\partial}{\partial r} \left( r A_{\theta} \right) = - \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta}. \]

The planes \( \phi(r, \theta, t) = \text{const.} \) - or, more precisely, their meridional cross-sections - determine the magnetic field lines. We shall consider the central body to be surrounded by a highly conducting gas whose motion is described by the equations of magnetohydrodynamics. For axially symmetric conditions these equations will assume the form
\[ \frac{d \phi}{dt} = \nabla \phi + \nabla \nabla \phi = \frac{\partial \phi}{\partial t} + v_r \frac{\partial \phi}{\partial r} + v_\theta \frac{\partial \phi}{\partial \theta} = 0, \]
\[ \frac{\partial \phi}{\partial t} = - \text{div} \mathbf{q} \mathbf{v}, \]
\[ \frac{\partial \phi}{\partial t} + (\nabla \nabla) \mathbf{v} = - \frac{1}{r} \nabla \rho + \frac{j_\phi}{c r \sin \theta} \nabla \phi, \]

where
\[ 4 \pi \frac{c}{j_\phi} = (\text{rot} \mathbf{H})_\phi = - \frac{1}{r \sin \theta} \frac{\partial^2 \phi}{\partial \theta^2} - \frac{\partial}{\partial \theta} \left( \frac{1}{r \sin \theta} \frac{\partial \phi}{\partial \theta} \right); \]
\[ j = (\mathbf{i}, \mathbf{j}, j_\phi) - \text{the central current density, } \mathbf{v} = (v_r, v_\theta, 0) - \text{the velocity of the gas and } \rho \text{ and } p - \text{the density and gas pressure. Taking as units of length, time, velocity, 'current function', density and pressure the characteristic values} \]
\[ L, \tau, V, \phi_0, \rho_0, p_0 \]
we write Equation (3) in the dimensionless form
\[ \frac{\partial \phi}{\partial t} = - \delta \nabla \nabla \phi, \]
\[ \frac{\partial \phi}{\partial t} = - \delta \text{div} \mathbf{q} \mathbf{v}, \]
\[ \frac{\varepsilon^2 \frac{\partial \phi}{\partial t}}{\varepsilon^2 (\nabla \nabla) \mathbf{v}} = - \delta \frac{\nabla \rho}{\rho} + \frac{j_\phi}{\rho r \sin \theta} \nabla \phi, \]

where the dimensionless parameters
\[ \delta = \frac{V \tau}{L}, \quad \varepsilon = \frac{V}{V_A}, \quad \delta^2 = \frac{p_0 \rho_0}{\rho_0 V_A}, \]

have been introduced, and
\[ V_A^2 = \frac{\phi_0^2}{4 \pi \rho_0 L^4} \]
denote the square of the Alfvén velocity.