PHYSICAL CONDITIONS IN THE CONVECTIVE
ENVELOPES OF STARS

I: Main-Sequence Stars

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Abstract. The physical conditions in the convective envelopes of Pop. II main-sequence stars are described with some detail. A particular care has been devoted to the effects due to the formation of hydrogen molecules and to the pressure ionization.

Riassunto. Vengono descritte in dettaglio le condizioni fisiche nelle zone convettive esterne delle stelle di sequenza principale di Popolazione II. In particolare vengono studiati gli effetti dovuti alla formazione della molecola dell'idrogeno ed alla ionizzazione per pressione.

1. Introduction

The stars with effective temperature lower than about $10^4$ degrees possess convective envelopes whose extension increases with decreasing surface temperature.

The radius of stellar models depends rather critically on the particular treatment of convection; unfortunately up to now no completely satisfactory theory of non-adiabatic convection has been developed. The mixing-length theory is the best we can use in constructing stellar models.

In this paper we discuss with some detail the physical structure of the convective envelopes of population II main-sequence stellar models obtained with the aid of the mixing length theory (CASTELLANI and RENZINI, 1968).

The chemical composition adopted in the models is $X=0.9$, $Y=0.099$. COX-STEWART (1965) opacities are employed; density scale height is used as mixing length in order to avoid the unphysical density inversion.

Other similar papers will follow pertinent to red giants branch and horizontal branch stars.

2. Thermodynamical Quantities

Several thermodynamical quantities play an essential role in the treatment of non-adiabatic convection, namely: The adiabatic gradient $V_a$, the specific heat $C_p$, the velocity of sound, the quantities which are needed to construct the density scale height. For all these quantities we used the formulae given by IBEN (1963) (1965) which include the contribution of the $H_2$ molecule and which are the most accurate at
present available. Since these quantities involve the concentrations of the various ionic species, the treatment of the ionization and dissociation equilibria turns to play an important role in determining the physical structure of the convective envelopes.

A. DISSOCIATION OF THE $H_2$ MOLECULE

The dissociation parameter of the $H_2$ molecule is taken from VARDYA (1961). The quantities $\xi_{AA}$ and $-(1/k) \left( \frac{dD}{dT} \right)$ involved in the Iben's formulae are then given by the expressions:

$$\xi_{AA} = \theta \left[ 11.34058 + \theta \left( 0.02257450 - 0.258769 \right) \right], \quad (1)$$

$$\frac{1}{k} \frac{dD}{dT} = 2.5 - 0.258770^2 + 0.045149\theta^3, \quad (2)$$

where $\theta = 5040.4/T$.

B. THE IONIZATION EQUILIBRIUM

It is well known that under conditions of local thermodynamical equilibrium the ionization equilibrium is governed by the Saha's equations

$$\frac{N_{i,j+1} C_e}{N_{i,j}} = A \frac{u_{i,j+1}}{u_{i,j}} \frac{T^{2.5}}{P} \exp \left( - \chi_{i,j} / kT \right) = K_{i,j} \quad (3)$$

where $N_{i,j}$ is the concentration by number of the atom $i$ in the $j$ stage of ionization, $C_e$ is the electron concentration, $u_{i,j}$ is its partition function and $\chi_{i,j}$ is its ionization potential; $A$ is a constant of the order of unity.

It is usual to set $u_{i,j}$ equal to the statistical weight of the ground level: this is a good approximation (for main-sequence stars) only for $M \geq 1.0 \, M_\odot$ but for smaller masses this approximation no longer holds and it leads to physical difficulties. The factor $T^{2.5}/P$ which appears in $K_{i,j}$ decreases continuously towards the interior of the star in all the regions where the effective temperature gradient is smaller than 0.4; in practice $\nabla_{\text{eff}} < 0.4$ everywhere except, eventually, in a narrow region of strongly superadiabatic convection. Since the exponential in Equation (3) approaches unity towards the interior, its influence on the ionization equilibrium vanishes progressively. It follows that after the saturation of the exponential ($\exp(-\chi_{i,j} / kT) \sim 1$) $K_{i,j}$ is proportional to $T^{2.5}/P$, i.e.: $K_{i,j}$ begins to decrease towards the interior leading to a decreasing ionization. This means that the assumption on the partition function is no more satisfactory.

If the quantity $T^{2.5}/P$ is very large at the bottom of the atmosphere, then complete ionization is attained before the saturation of the exponentials and the difficulty represented by the receding ionization can be avoided by simply stopping the computation of ionization equilibrium when the ionization is practically complete. In practice this is the case for $M \geq 1.0 \, M_\odot$ (on the main sequence). But for $M < 1.0 \, M_\odot$, $K_{i,j}$ begins to decrease when the ionization is far to be complete; in this case the only way to obtain a good representation of the physical situation is to evaluate the partition functions.