Abstract. Recently some authors have questioned whether Newton's law of gravitation is actually true on scales less than 1 km. The available constraints on the gravitational constant show that its laboratory value $G_0$ may differ from the value at infinity $G_\infty$ by $\sim 40\%$. Long (1976) reported experimental evidence for departures from Newton's law. In this note it is shown that the difference between $G_0$ and $G_\infty$ modifies the mass–radius relation of degenerate stars. The observations of white dwarfs are consistent with the theory of stellar evolution only if $G_0$ differs from $G_\infty$ by not more than $\sim 10\%$. This estimate may be improved by a higher accuracy of observations.

The scalar gravity with nonzero rest mass leads to the Yukawa-type term (in the weak field approximation) in addition to the Newtonian potential. This term modifies Newton's law to the form

$$F_{grav} = G(r)Mm/r^2$$

with

$$G(r) = G_\infty[1 + \alpha(1 + \mu r)\exp(-\mu r)],$$

where $\mu^{-1}$ may be interpreted as the Compton length of a massive graviton, $G_\infty$ denotes the gravitational constant for $\mu r \gg 1$, and the parameter $\alpha = \frac{1}{3}$ according to Fuji (1972) and O'Hanlon (1972). Space variability of $G$ has also been discussed by Wagoner (1970), Ulrich (1974) and Long (1976). Mikkelsen and Newman (1977) have used celestial mechanics, models of the Earth and the Sun, to show that if $10 \text{ m} < \mu^{-1} < 1 \text{ km}$, then $G_\infty$ may differ by $40\%$ from the laboratory value $G_0 = 6.67 \times 10^{-8} \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-2}$. This is not sufficient to rule out the value $\alpha = \frac{1}{3}$ in Equation (1), which gives $G_\infty = 0.75 G_0$.

In this note it is shown that the difference between $G_\infty$ and $G_0$ changes the theoretical mass–radius relation $M(R)$ for degenerate stars. It is argued that observations of the most extensively studied white dwarfs make it clear that a difference between $G_\infty$ and $G_0$ by as much as $30\%$ is inconsistent with the generally accepted predictions of the theory of stellar evolution.

For $10 \text{ m} < \mu^{-1} < 1 \text{ km}$ the structure of a white dwarf with a typical radius of $10^4 \text{ km}$ is determined by the usual equations, where $G_\infty$ is substituted for $G_0$. For a cold star with any barotropic equation of state $P = P(\rho)$, the similarity relations give

$$R = R_0(G_0/G_\infty)^{1/2}, \quad M = M_0(G_0/G_\infty)^{3/2},$$

where $M_0(R_0)$ is the mass–radius relation determined for $G = G_0$. For stars, the

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observable quantity is not $M$ (in grams) but $G \cdot M$; and the stellar mass is usually measured in solar units ($M_\odot$). Since, for the Sun, the observed quantity is also $G_\odot M_\odot$, for a degenerate star we obtain $M/M_\odot \propto G_\odot^{-1/2}$.

The relations (2) were first used to derive the constraints on $G$ by Sugimoto (1972); Fujimoto and Sugimoto (1972) were the first also to use neutrino observations of the Sun for the same purpose.

The solid lines in Figure 1 show the relations $M(R)$ for cold white dwarfs with Salpeter's (1961) equation of state for $G_\infty = 0.75 G_0$. The curves are obtained from the calculations of Hamada and Salpeter (1961) by transformation (2). The observed positions of two of the most extensively studied white dwarfs, Sirius B and 40 Eri B, are also plotted.

Let us first concentrate on 40 Eri B. Its mass was determined by Heintz (1974) as $M = (0.43 \pm 0.02) M_\odot$ and the radius by Matsushima and Terashita (1969) as $R = (1.50 \pm 0.03) \times 10^{-2} R_\odot$ and by Shipman (1972) as $R = (1.32 \pm 0.06) \times 10^{-2} R_\odot$. The value obtained by Shipman seems more reliable since it is derived from absolute multichannel measurements (Oke, 1974). Both values, and even a compromise value of $R = (1.4 \pm 0.1) \times 10^{-2} R_\odot$ (Liebert, 1976), show that the position of 40 Eri B is inconsistent with the $^{12}$C-curve. At the same time a simple evolutionary argument shows that 40 Eri B should not consist of elements heavier than $^{12}$C/$^{16}$O (the line for oxygen in Figure 1 is almost the same as for $^{12}$C), and the iron composition suggested by Figure 1 is definitely excluded. It can be put as follows.

![Fig. 1. The observed positions of white dwarfs on the mass–radius diagram and theoretical relations $M(R)$ for $G_\infty \neq G_0$. MT is the radius of 40 Eri B found by Matsushima and Terashita (1969) and S as found by Shipman (1972).](image-url)