THE GENERATION OF HIGH-FREQUENCY GRAVITATIONAL WAVES IN A PERFECT FLUID

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(Received 25 August, 1983)

Abstract. The problem of the generation of a gravitational wave by an acoustic one is studied in the high-frequency domain for a perfect fluid. The negative result which is found may be of some relevance for astrophysical and cosmological applications.

1. Introduction

It is well known that Einstein’s equations have solutions which can be interpreted as gravitational waves.

The experimental search for such waves has been one of the main challenges of the last years. Even if at present this search has not been successful, yet, there is a strong effort underway towards improving the sensitivity of gravitational wave detectors (Tyson and Giffard, 1978).

On the other hand from the theoretical point of view some fundamental problems still remain to be clarified regarding the possible astrophysical sources and the mechanism of generation of gravitational radiation.

Roughly speaking we can distinguish two astrophysical situations where generation of gravitational waves can occur; in one case we can have a quiet emission from a system in a quasi-stationary case, in the other a violent burst from abruptly changing sources.

In both cases, since the gravitational luminosity varies as $(M/R)^5$ one looks for emission from very compact and massive objects. In the case of quiet emission one therefore looks for gravitational waves from a rapidly rotating neutron star (Bertotti and Anile, 1973) or from close binary systems with one or more compact component like PSR 1913 + 16 for which there is some observational evidence of gravitational radiation (Hulse and Taylor, 1975; Demiansky and Shakura, 1976). In the case of violent emission one envisages situations such as anisotropic stellar collapse to a black hole, the fall of ‘debris’ onto an already formed black hole, supernova explosions.

The greatest theoretical difficulty encountered when assessing the gravitational flux from possible sources lies in the nonlinear character of Einstein’s equations and, for this reason, approximate methods are usually employed.

In this article we shall tackle the problem of gravitational radiation generated by acoustic waves. This can be of interest in stellar collapse and galaxy formation, where acoustic waves play an important role.
2. Notations and Conventions

Latin indices $a$, $b$, ... generally run over the four space-time coordinate labels 0, 1, 2, 3, greek indices $\mu$, $\nu$, ... run over the three space coordinate labels 1, 2, 3.

The metric signature is taken to be $+2$. Symmetrization of indices is indicated by $( )$, antisymmetrization by $[ ]$. Ordinary derivatives are indicated by $\partial_a$ or by a comma, covariant derivatives by $\nabla_a$ or by a semi colon.

3. Linear Perturbations

Let $g_{ab}$ and $T_{ab}$ be the metric and energy-momentum tensor of a space-time $\mathcal{M}$.

Now let us perturb the covariant components of the metric and the energy-momentum tensor by writing

\[
g_{ab} = g_{ab} + \varepsilon h_{ab},
\]
\[
T_{ab} = T_{ab} + \varepsilon \tilde{T}_{ab} + O(\varepsilon^2),
\]

where $\varepsilon$ is a small parameter.

The well-known perturbed Einstein equations read (Weinberg, 1972)

\[
\frac{1}{2} g^{ab} \left[ h_{ab; c; d} - h_{bc; d; a} - h_{bd; c; a} + h_{cd; b; a} \right] =
- \left( \tilde{T}_{cd} - \frac{1}{2} g_{cd} \tilde{T}_{bp} - \frac{1}{2} g_{cd} h^{as} T_{as} - \frac{1}{2} h_{cd} T_s \right),
\]

(2)

together with these we have to take account the perturbed conservation equations which are of the form

\[
\tilde{T}^{ab}_{;b} + \tilde{T}^{cb}_{a} T^{ac} + \tilde{T}^{a}_{cb} T^{cb} = 0,
\]

(3)

where $\tilde{T}^{ab}_{bc}$ are the perturbations of the Christoffel symbols

\[
\tilde{\Gamma}^a_{bc} = \Gamma^a_{bc} + \varepsilon \tilde{\Gamma}^a_{bc} + O(\varepsilon^2)
\]

(4)

with

\[
\tilde{\Gamma}^a_{bc} = \frac{1}{2} g^{ar} \left[ h_{rb; c} + h_{rc; b} - h_{bc; r} \right]
\]

(5)

and all the covariant derivatives are performed with respect to the unperturbed metric $g_{ab}$.

$h_{ab}$ and $T_{ab}$ are determined up to the gauge transformations

\[
'h_{ab} = h_{ab} + \mathcal{L}_{\xi} g_{ab},
\]
\[
'\tilde{T}_{ab} = \tilde{T}_{ab} + \mathcal{L}_{\xi} T_{ab},
\]

(6)

(7)

where $\xi$ is an arbitrary vector and $\mathcal{L}_{\xi} g_{ab}$, $\mathcal{L}_{\xi} T_{ab}$ are the Lie derivatives, given by

\[
\mathcal{L}_{\xi} g_{ab} \equiv \xi_a; b + \xi_b; a,
\]
\[
\mathcal{L}_{\xi} T_{ab} \equiv T_a \xi_c; b + T_b \xi_c; a + T_{ab; c} \xi^c.
\]

(6a)

(7a)