SOME GENERAL RESULTS IN SELF-CREATION COSMOLOGIES

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Abstract. Raychaudhuri-type equations are written for a cosmological model filled with perfect fluid and obeying the equations of a self-creation theory recently proposed by G. A. Barber. In addition some general results on spatially homogeneous cosmological models are obtained. The Hawking-Penrose energy condition suggests that the singularity can be avoided in this theory.

1. Introduction

Many theories have been proposed as alternatives to Einstein's theory. The most important among them being the scalar-tensor theory proposed by Brans and Dicke (1961). This theory develops Mach's principle in a relativistic framework by assuming that inertial masses of fundamental particles are not constants, but are dependent upon the particles' interaction with some cosmic scalar field coupled to the large-scale distribution of matter in motion. The Brans-Dicke theory does not allow the scalar field to otherwise interact with fundamental particles and photons. By allowing the scalar field to interact with particle and photon momentum 4-vectors and thus modifying the Brans-Dicke theory to allow for the continuous creation of matter Barber (1982) has developed a continuous creation theory. In this theory the Universe is seen to be created out of self-contained gravitational, scalar and matter field. The theory predicts local effects that are within the limits already observed. In the limit \( \lambda \to 0 \) the theory approaches to standard general relativity (GR) theory in every respect.

In this note we have written Raychaudhuri-type equations in this theory and have proved a general result for spatially homogeneous stationary cosmological models. Hawking-Penrose energy conditions have also been discussed.

2. Homogeneous Cosmological Models in the Self-Creation Theory

The field equations in the theory proposed by Barber (1982) are

\[
R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R = - \frac{8 \pi}{\phi} T_{\mu \nu} - \frac{2}{3 \phi \lambda} \phi_{; \mu ; \nu} + \frac{2}{3 \phi \lambda} g_{\mu \nu} \Box^2 \phi
\]

(1)

and

\[
\Box^2 \phi = ( - g )^{-1/2} [ ( - g )^{1/2} \phi^a ]_{,a} = 4 \pi \lambda T_{\mu \nu}^\phi,
\]

(2)

where $\Box^2 \phi = \phi_{\alpha ; \alpha}$ is the invariant d'Alembertian and the contracted tensor $T^\mu_\nu,\rho$ is the trace of energy momentum tensor describing all non-gravitational and non-scalar field matter and energy; $\lambda$ is coupling constant chosen so that $G$, which is now a function of $\phi$ can be defined as $G = 1/\phi$.

Finding the trace of Equation (1) and using (2), we get $R = 0$. Therefore, Equation (1) reduces to

$$R_{\mu\nu} = -\frac{8\pi}{\phi} T_{M\mu\nu} - \frac{2}{3\phi \lambda} \phi_{\mu ; \nu} + \frac{2}{3\phi \lambda} g_{\mu\nu} \Box^2 \phi. \quad (3)$$

Now we take the energy momentum tensor of a perfect fluid as

$$T_{M\mu\nu} = (p + \rho) u^\mu u_\nu - \rho g_{\mu\nu} \quad (4)$$

with restrictions $\rho > 0$ and $p > 0$. We have in this theory

$$T^\mu_\nu,\mu = \frac{2 - 3\lambda}{6} (T^\mu_\rho ; \rho) - \frac{(2 - 3\lambda)}{24\pi \lambda} \Box^2 (\phi, \nu). \quad (5)$$

From Equations (4) and (5) we have

$$\dot{u}_\nu = \frac{p_{,\mu} h^\mu_\nu}{(\rho + p)} + \frac{(2 - 3\lambda)(\rho - 3p)}{6(\rho + p)} - \frac{(2 - 3\lambda) \Box^2 (\phi, \nu)}{24\pi \lambda (\rho + p)} \quad (6)$$

and

$$\theta = -\frac{(4 + 3\lambda)\rho}{6(\rho + p)} - \frac{(2 - 3\lambda)\dot{\rho}}{2(\rho + p)} - \frac{(2 - 3\lambda) \Box^2 (\phi)}{24\pi \lambda (\rho + p)}, \quad (7)$$

where $\dot{u}_\nu = u_{,\mu} u^\mu$ and in the following the dot will indicate the covariant derivative along the world line $h^{\mu\nu}$ and $\theta$ are projection tensor and the scalar of expansion, respectively, defined as

$$h^{\mu\nu} = g^{\mu\nu} - u^{\mu} u^{\nu}, \quad (8)$$

$$\theta = u^{\nu}_{,\nu}. \quad (9)$$

We may introduce $l$ by these equation

$$\frac{1}{3} \theta = \frac{l}{l}. \quad (10)$$

Substituting from (4) into (3) we have

$$\phi_{\alpha ; \alpha} = 4\pi \lambda (\rho - 3p). \quad (11)$$

We may write Equation (11) in the form

$$[\phi^3(-g)^{1/2}]_{,\alpha} = 4\pi \lambda (\rho - 3p) (-g)^{1/2}. \quad (12)$$