UNSTEADY FLOW OF A NON-NEWTONIAN VISCO-ELASTIC FLUID BETWEEN TWO COAXIAL CIRCULAR CYLINDERS

KRISHNA RAJ CHOUBEY

Department of Mathematics, Banaras Hindu University, Varanasi, India

(Received 19 March, 1984)

Abstract. In this paper, the flow of a visco-elastic liquid between two coaxial circular cylinders has been studied when inner cylinder is moving from rest for a certain period with linearly growing speed and then stops suddenly. The Laplace transform technique has been employed to solve the basic differential equations. The expression for the velocity field is obtained.

1. Introduction

Flow of both Newtonian and non-Newtonian liquids through pipes and concentric cylinders has wide engineering applications and has attracted the attention of many authors. Due to the non-linearity of the differential equations in the theory of elastico-viscous liquids of Rivlin and Ericksen (1955) type, only a few problems with a high degree of symmetry have been solved in an exact way. For a purely viscous liquid there exists a wealth of literature. The steady-state laminar flow of an incompressible viscous liquid in an annulus with coaxial impermeable walls rotating about a common axis has been considered in the book by Schlichting (1962). Oldroyd (1950) and Walters (1960) have solved similar problems with an elasto-viscous liquid in the annulus. Pipkin (1964) and Jones and Walters (1967) have examined the flows of certain types of non-Newtonian fluids.

More recently, Teipel (1981) has studied the impulsive motion of a flat plate in a visco-elastic fluid (Rivlin–Ericksen type). The aim of this work is to study the unsteady motion of an Rivlin–Ericksen elastico-viscous fluid through the annulus between two concentric circular cylinders. The motion is caused by the inner cylinder moving from rest at $t = 0$ along its axis with velocity linearly increasing with $t$, for a finite period and then coming to rest at $t = T$. This problem is of importance because this type of motion corresponds to the case of piston movement. Laplace transform technique have been employed to solve the basic differential equations.

2. Basic Equations

The second-order approximation of the general constitutive equation given by Rivlin and Ericksen (1955) can be written in the following form (cf. Astarika and Marucci, 1974)

$$T = -pI + \mu A_1 + \gamma A_1^2 + \beta A_2,$$

(1)

where $T$ is the stress tensor; $p$, the pressure; $I$, the unit tensor; $A_1$ and $A_2$, the first two
Rivlin–Ericksen tensors; and $\mu$, $\alpha$, and $\beta$ are three material constants. Moreover, $A_1$ and $A_2$ are given by symmetrical matrices defined by

$$A_1 = \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i},$$

$$A_2 = \frac{\partial}{\partial x_j} \left( \frac{D v_i}{Dt} \right) + \frac{\partial}{\partial x_j} \left( \frac{D v_j}{Dt} \right) + 2 \frac{\partial v_m}{\partial x_i} \frac{\partial v_m}{\partial x_j}, \quad (i, j, m = 1, 2, 3). \quad (2)$$

The equations of motion and continuity are given by

$$\rho \left( \frac{\partial v^i}{\partial t} + v^j \frac{\partial v^i}{\partial x_j} \right) = -p_{,j} + p^j_{,j} \quad (3)$$

and

$$v^j_{,i} = 0 . \quad (4)$$

We consider the unsteady motion of an elasto-viscous incompressible liquid through an annular region formed by two infinitely long concentric cylinders of radii $a$ and $b$ ($a > b$), the inner cylinder moves with axial velocity $W$, relative to the stationary outer one. Thus, it starts moving at $t = 0$ with linearly increasing velocity in the axial direction and stops at $t = T$.

Let us choose the cylindrical polar coordinates $(r, \theta, z)$, the axis of $z$ coinciding within the common axis of the cylinders and let $(v_r, v_\theta, v_z)$ be the components of the velocity of the fluid in the respective directions of $(r, \theta, z)$ increasing. We assume that the flow is axisymmetric and the cylinders are of infinite length, so that the physical quantities are independent of $z$ and $\theta$. In fact, consistent with the continuity equation, we may take

$$v_r = 0 , \quad v_\theta = 0 , \quad v_z = w(r, t) . \quad (5)$$

Since the flow takes place due to the shearing action of the inner cylinders and from Equations (1)–(5), the surviving stress components are

$$\rho \frac{\partial w}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( r p'_{rz} \right) , \quad (6)$$

where $p'_{rz}$ is given by

$$p'_{rz} = \mu \left[ \frac{\partial w}{\partial r} + \beta_0 \frac{\partial^2 w}{\partial t \partial r} \right] . \quad (7)$$

The boundary conditions are

$$w(r, t) = 0 \quad \text{for} \quad t \leq 0 , \quad b \leq r \leq a ,$$

$$w(b, t) = W [u(t) - u(t - T)] , \quad t > 0 ,$$

$$w(a, t) = 0 , \quad t \geq 0 ;$$

where $u(t)$ is the Heaviside unit step function.