HYPERONIC EQUATION OF STATE

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(Received 4 August, 1969)

Abstract. An equation of state for cold matter at neutron star densities, \( \rho > 10^{14} \text{ gm/cm}^3 \), is evaluated. The gas is considered to be a degenerate mixture of neutrons, protons, leptons, hyperons and massive baryons. We derive the equilibrium equations including the effects of nuclear interactions among all the hadrons.

1. Introduction

The purpose of this work is to calculate an equation of state for the interior of stars in which the densities exceed that of nuclear matter. This is essential to the calculation of the maximum mass neutron star that may be formed after a supernova explosion. In a previous paper (Langer et al., 1969) an equation of state was developed for densities up to \( 1.0 \times 10^{14} \text{ gm/cm}^3 \). This was accomplished by including nuclear interactions and minimizing the relevant thermodynamic potentials. When one extends the method of calculating the equation of state beyond this point, certain complexities enter the problem. Due to the combination of nuclear interactions plus fermi level of the baryon, the effective mass of the neutrons increases with increasing density. A point is reached, therefore, where the chemical potential of the neutron exceeds the rest mass of the sigma minus hyperon – a mass of 1197 MeV (Table I). From this point on, hyperons (and other massive baryons) enter as components of the equation of state. Each hyperon in turn interacts with all other baryons and must satisfy its own set of equilibrium conditions, including, of course, conservation of charge and baryon number. One is hurt by two principal problems: on one side is the necessity to use a proper nuclear interaction and on the other is the increasing numerical difficulty which goes almost exponentially with the addition of new hyperons as the density increases. The latter problem enters because of the requirement of solving a set of coupled equations for all components in the equation of state. These are in reality integral equations since the binding per particle must be found by integrating the contribution of nuclear interactions due to all other particles. Tsuruta and Cameron (1966) developed an approximate treatment for a hyperon equation of state by including nuclear interactions only after the equilibrium number densities of non-

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interacting particles in the gas had been found. In this paper, the full set of coupled equations is solved, using of course a good fast computer.

The nuclear interaction used in this work is that developed by Weiss and Cameron (1969) based upon the Levinger-Simmons $V_a$ and $V_y$ potentials. Levinger and Simmons (1961) originally developed these potentials from neutron scattering data between 20 and 340 MeV. A word of caution must therefore be entered. The equation of state extends in the upper region beyond this energy for some of the constituents. The maximum energy of any of the constituents never tends to more than 410 MeV, however, since the equation of state is cut off at the point at which the pressure equals the energy density. This limit represents the requirement that the speed of sound shall never exceed the speed of light (Zeldovich, 1962).

### 2. Equation of State

We have to find the equation of state for a mixture of leptons and baryons where nucleon–nucleon interactions are included (electrostatic interactions are neglected because their energies are orders of magnitude smaller than the other interaction energies). The equation of state will follow from the conditions which determine the equilibrium mixture of the gas. These conditions will follow from minimizing the thermodynamic potential

$$\Phi = \Phi(P, T, n_i),$$

where $T$ is the temperature, $P$ the pressure and $n_i$ the number densities for particle $i$. $\Phi$ will be minimum for fixed $T (=0 \text{ K})$ and $P$ when $d \Phi = 0$, or

$$\frac{\partial \Phi}{\partial n_j} + \sum_i \frac{\partial \Phi}{\partial n_i} \frac{\partial n_i}{\partial n_j} = 0$$

for all $j$.  

### Table I

<table>
<thead>
<tr>
<th>Particle</th>
<th>Mass (MeV)</th>
<th>Charge ($q$)</th>
<th>Spin ($s$)</th>
<th>Threshold density ($10^{14} \text{ gm/cm}^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu^-$</td>
<td>105.7</td>
<td>$-1$</td>
<td>$\frac{1}{2}$</td>
<td>7.75, 2.30, 2.35</td>
</tr>
<tr>
<td>$\Sigma^-$</td>
<td>1197.0</td>
<td>$-1$</td>
<td>$\frac{1}{2}$</td>
<td>11.2, 2.60, 2.83</td>
</tr>
<tr>
<td>$\Lambda^0$</td>
<td>1115.0</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>19.1, 4.02, 4.17</td>
</tr>
<tr>
<td>$\Sigma^0$</td>
<td>1197.0</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>70.6, 9.28, 8.93</td>
</tr>
<tr>
<td>$\Lambda^0$</td>
<td>1236.0</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>112.0, 12.1, 11.6</td>
</tr>
</tbody>
</table>