Abstract. Differential equations governing the dynamical tides in close binary systems consisting of centrally condensed components of viscous gas are split up (Section 2) in their real and imaginary parts, the ratio of which defines the tidal lag. In Sections 3 and 4 these equations will be particularized to a case in which the central mass-point of each star is surrounded by an evanescent envelope the density of which decreases as the inverse square of the central distance. It is shown that self-gravitating configurations built up in accordance with this model are incapable of performing free non-radial oscillations with a frequency \( \omega \) comprised between \( 0 \leq \omega^2 \leq \infty \); but explicit expressions for forced oscillations representing dynamical tides are given for an arbitrary form of the external field of force. Equations for the imaginary components of the displacement, constructed for the same model in Section 4, disclose that if the viscosity of stellar material is identified with that of hydrogen plasma, the tidal lag due to a viscous dissipation of kinetic energy may produce dynamical effects, the cumulative outcome of which becomes appreciable on the Kelvin time-scale, but over short intervals of time their stationary photometric effects should be negligible. The latter can become observationally significant only for stars in which turbulent viscosity under near-adiabatic conditions becomes an important factor.

1. Introduction

In the preceding papers of this series (cf. KOPAL, 1968a, b, c, hereafter referred to as Papers I, II and III, respectively) linearized equations were set up which govern the dynamical tides in close binary systems the components of which consist of viscous gas; and by the neglect of the oscillations in gravitational potential reduced (cf. Paper III) to the case of centrally condensed configurations. The purpose of the present investigation will be to re-trace our steps to the fundamental equations of Paper I defining the (generally complex) velocities of tidal displacements governed by them in their real and imaginary parts, and to construct their solutions for centrally condensed models which should define both the amplitudes of the respective tides, as well as their displacement in phase (i.e., tidal lag).

The concept of a 'tidal lag' in close binary systems has been invoked many years ago – for the first time, I believe, by DUGAN (1920) in his study of the eclipsing system of U Cephei – to account for the observed asymmetries of the light curves of this and other eclipsing variables between minima; and later it found a significant place in the fertile speculations of Otto Struve (cf., e.g., STRUVE, 1950). Yet all these early views were implicitly based on a mistaken analogy with the behaviour of dynamical systems consisting of rigid bodies. If the distorted components of close binaries could – for dynamical purposes – be regarded as rigid, their semi-major axes could indeed librate about the line joining the centres of the two bodies with arbitrary amplitudes; and only the periods of such librations would be constrained by the properties of each
respective system (cf. Walter, 1931, 1933). However – unlike the solids – gaseous (and, therefore, almost perfectly fluid) bodies cannot perform free lateral oscillations about their positions of equilibrium in an external field of force.

Instead, their dynamical tides constitute forced oscillations, which can become asymmetric with respect to the direction of the attracting force only to the extent permitted by the presence of dissipative phenomena – such as arising from viscosity. Contrary to other opinions then prevailing, the present writer pointed out already a quarter of a century ago, in his study of the Algol system (Kopal, 1942) or U Cephei (1944), that the actual viscosity in the outer layers of these stars is unlikely to make any tides lag by more than a small fraction of a degree, and thus give rise to no observable asymmetry of their light changes. It is not, however, till at the present time that an adequate theory has been developed to provide a basis for making such statements both general and quantitative. The main conclusions arrived at in the course of this study have already been summarised in the abstract; so that in what follows our task will be to substantiate them in detail.

2. Equations of the Problem

In order to develop the mathematical basis of our problem, let us depart from Equations (11) and (12) of Paper II, and split up their radial velocity-components \( u, v \) in their real and imaginary parts by setting

\[
\begin{align*}
\begin{cases}
  u = u_1 + iu_2 \\
  v = v_1 + iv_2
\end{cases}
\end{align*}
\]

(1)

in accordance with Equations (81) of the same paper, where \( i \) stands for the imaginary unit. The time-derivatives \( \dot{C}_{i,j} \) of the coefficients of expansion of the external potential are real quantities; but with complex velocities \( u \) and \( v \) the quantities \( \overline{\zeta}, \phi \) arising from viscosity and defined by Equations (8)-(9) of Paper II become likewise complex, and expressible by

\[
\begin{align*}
\overline{\zeta} &= \zeta_1 + i\zeta_2, \\
\phi &= \phi_1 + i\phi_2
\end{align*}
\]

(2)

where \( \zeta_{1,2} \) as well as \( \phi_{1,2} \) are real. If, moreover, \( y \) as defined by (13) of Paper II is likewise decomposed into

\[
y = y_1 + iy_2,
\]

(3)

the system of Equations (11)-(12) of Paper II can – by separation of their real and imaginary parts – be replaced by four simultaneous equations of the form

\[
\frac{\partial}{\partial r} \left\{ \rho \left( a^2 y_1 - gu_1 \right) \right\} + g \left\{ \rho y_1 + u_1 \frac{\partial \rho}{\partial r} \right\} + \rho \sigma^2 u_1 - \sigma \overline{\zeta}_{2} + j\sigma \rho^{-1} \dot{C}_{i,j} = 0,
\]

(4)

\[
\frac{\partial}{\partial r} \left\{ \rho \left( a^2 y_2 - gu_2 \right) \right\} + g \left\{ \rho y_2 + u_2 \frac{\partial \rho}{\partial r} \right\} + \rho \sigma^2 u_2 + \sigma \overline{\zeta}_{1} = 0;
\]

(5)