A SEMANTIC ANALYSIS
OF CONDITIONAL ASSERTION*

1. INTRODUCTION

By the term 'a conditional assertion' we mean, following Belnap, a sentence of the form If A then B' with the reading that if A is true, B is asserted, and nothing is asserted otherwise. In other words, a person expressing the conditional above is taken to have committed himself to the truth of B in case A is true, and he has not committed himself to anything if A is not true.

The aim of this paper is to develop a semantical analysis of conditional assertion along the lines suggested by Belnap [1] and [2] using Bressan's approach in [3]. The consequential analysis enables us both to generalize some of Belnap's results and to obtain a language sufficiently rich containing various different modal operators and definite descriptions which enable us to discuss other conditional forms and their applications.

2. THE LANGUAGE CA' FOR CONDITIONAL ASSERTION

Following Bressan, we consider a v-sorted modal language CA' for v=1, 2,..., CA' has various wffs which are either matrices or terms. There are v sorts of individual terms to which we refer to as wffs of types 1, ... v. The types of wffs of CA' are defined as follows:

DEFINITION 1. t is a termttype for CA', briefly t∈τ' is defined recursively as follows:

(i) If t is any of the integers 1, ..., v then t∈τ'
(ii) If t₁,..., tₙ∈τ' then (t₁,..., tₙ)∈τ' (called an attribute type).
(iii) If t₁,..., tₙ, t₀∈τ' then (t₁,..., tₙ; t₀)∈τ' (called a function type).

Let τ⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻˓
The signs of our language $CA'$ are the connectives: $\sim$ (negation), $\&$ (conjunction), $\rightarrow$ (implication), $\langle \rangle$ (conditional); the universal quantifier $\langle x \rangle$, the equality sign $=$, the (categorical) necessity sign $\Box$, the (sentential) necessity sign $\square$, the (propositional) necessity sign $N$, the iota operator $i$, left and right parentheses (‘(’ and ‘)’), the comma ‘,’ the variables $v_n$ and constants $c_n$ (where $t \in \tau^{-}$, $n = 1, 2, \ldots$). We say that $v_n$, $c_n$ are variables and constants respectively of type $t$ and index $n$. We presuppose that the connectives $\lor$ (disjunction), $\Rightarrow$ (material implication), $\equiv$ (material equivalence), $\leftrightarrow$ (propositional equivalence), the existential quantifier $\exists$ and the three possibility signs $\diamond$, $\downarrow$ and $M$ (the duals of the three necessity signs) are defined in the usual way.

The $\text{wffs}$ of $CA'$ are defined to be the typed expressions of $CA'$, i.e. $F$ is a wff of $CA'$ iff there is a $t \in \tau^{-}$ such that $F \in E_t$, where:

**DEFINITION 2.** For $t \in \tau^{-}$ we define the formula $F$ has the type $t$, in short $F \in E_t$, recursively by the following conditions (where $n$ runs over positive integers).

1. $v_n \in E_t$ and $c_n \in E_t$ for $t \in \tau^{-}$.
2. If $t, F_1, F_2 \in E_t$ and $F$ is $F_1 = F_2$, then $F \in E_0$.
3. If $t_1, \ldots, t_n, F_1, F_n \in E_{t_1}, \ldots, E_{t_n}$ and $R \in E_{(t_1, \ldots, t_n)}$ and $F$ is $R(F_1, \ldots, F_n)$, then $F \in E_0$.
4. If $t_1, \ldots, t_n, t_0 \in \tau^{-}, F_1, F_n \in E_{t_1}, \ldots, E_{t_n}$, $F_0 \in E_{t_0}$, then $\Phi(F_1, \ldots, F_n) \in E_0$.
5. If $F_1, F_2 \in E_0$ then $\sim F_1, F_1 \& F_2, (F_1/F_2), (F_1 \rightarrow F_2)$, $(v_n)(F_1), \Box F_1, \square F_1, N F \in E_0$.
6. If $t \in \tau^{-}, F_1 \in E_0$ and $F$ is $(v_m)F_1$, then $F \in E_t$.

We call the elements of $E_0$ matrices in $CA'$, and for $t \in \tau^{-}$ the elements of $E_t$ are terms in $CA'$ of type $t$. In particular, these terms are individual terms if $1 \leq i \leq v$, relators if $t$ has the form $(t_1, \ldots, t_n)$ and functors, if $t$ has the form $(t_1, \ldots, t_n : t_0)$, where $t_1, \ldots, t_n \in \tau^{-}$. Closed matrices are called sentences.

3. Semantics for $CA'$

In our semantics we use a universe which comprises a domain $D$ and a choice function $\alpha'$. The domain $D$ is a set of $v$ individual domains, $D = \{D_1, \ldots, D_v\}$. Each $D_t$ has at least two elements, one an ‘existent