RADIAL AND NONRADIAL PULSION OF A ROTATING PARTIALLY DEGENERATE STANDARD MODEL

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Abstract. The effect of slow uniform rotation on the radial and nonradial modes of partially degenerate standard models has been investigated. For the case of the radial mode, it is shown that the destabilizing effect of slow uniform rotation reduces with increased central condensation of the model. However, it is found that the models with a strongly degenerate interior, become more unstable dynamically as a result of slow uniform rotation. Further, it is noted that the frequency of the nonradial modes of oscillation (Kelvin mode) increases due to the presence of rotation. Thus the period of radial modes of oscillation of a slowly uniformly rotating partially degenerate standard model are much larger than the corresponding periods of nonradial (Kelvin) modes.

1. Introduction

Recent observations of ZZ Ceti variables show that some of them are pulsating with a single characteristic frequency while others have multiperiodic structures in their power spectrum (McGraw and Robinson, 1975; McGraw et al., 1981). Further, the observations on low-mass dMe stars and those by eond the spectral class F5 shows the presence of rotation and possibly magnetic fields (Petterson, 1983). From the evolutionary tracks of low-mass stars ($M \leq 0.5$) it is known that most of the interior becomes degenerate partially or completely either on reaching the Main Sequence or subsequent post-Main Sequence evolution towards white dwarfs of low masses. Theoretically, the results of observations on low-mass stars and white dwarfs poses considerable difficulties in their explanation. In the present investigation, we consider low-mass partially degenerate rotating standard models (Das, 1985a, b; hereafter referred to as Papers I and II, respectively) and study the effect of a small uniform rotation on their pulsation characteristics. The analysis is based on using the variational formulation of Clement (1965). Earlier, Chlebowski (1978) studied the effect of a small rotation on the g-modes of white dwarfs using the method of Simon (1969). We confine ourselves here to the study of the rotational effect on the radial and nonradial axisymmetric ($j = 2; m = 0$) modes. The radial and nonradial modes have been computed for models with different degree of central degeneracy. The models chosen here have a central condensation comparable to white dwarfs of masses somewhat below the Chandrasekhar limit. It has been noted that as the central condensation increases, the destabilizing effect of rotation, with regard to the dynamical stability of the model, reduces and in fact changes sign; that is, when an extremely large central degeneracy has developed in the model interior, a small rotation tends to make it dynamically more unstable. Thus the radial pulsation period of rotating models are in general larger than the periods of corre-
sponding nonrotating models. The rotational contribution to the frequency of nonradial (Kelvin mode) oscillation \((j = 2; m = 0)\) tends to increase with an increase in the central degeneracy. For polytropic stellar models with \(n \geq 3.0\), Clement (1965) showed that in the first approximation the change in the frequency of Kelvin's mode due to rotation is positive, while in the second approximation it is negative. However, we observed that in both the first and second approximation, the corrections to the frequency of Kelvin's mode remain positive in spite of an increase in central condensation (from approximately 12 to 35).

2. Basic Equations

The change \((\alpha_{2}^{2})\) in squared frequency of radial and nonradial modes of pulsation of a uniformly gaseous mass is given (cf. Clement, 1965) by

\[
-2\alpha_{2}^{2} \int \rho_{0} |\zeta|^{2} \, dx = \alpha_{0}^{2} \int \rho_{2} |\zeta|^{2} \, dx + \int \zeta \cdot L_{2}(\zeta) - 4 \int \rho_{0} \zeta w^{2} \, dx,
\]

where \(\alpha_{0}\) denotes the frequency of pulsation of the unperturbed model, \(\rho_{0}\) the unperturbed density, and \(\rho_{2}\) the contribution of rotation in changing the density function of \(\rho\). Further, the second term on the right-hand side of Equation (1) involves the contribution of rotation and its effects on the equilibrium structure of the model, and could be written as

\[
\int \zeta \cdot L_{2}(\zeta) \, dx = -\int (\gamma_{0}(x) - 2)P_{2}(\nabla \cdot \zeta)^{2} \, dx - \int \gamma_{2}(x)P_{0}(\nabla \cdot \zeta)^{2} \, dx +
\]

\[
+ 2 \int P_{2} \zeta \cdot \nabla (\nabla \cdot \zeta) \, dx + \int \frac{P_{2}}{\rho_{0}} \left[ \rho_{0}(\zeta \cdot \nabla \rho_{0}) (\nabla \cdot \zeta) + \rho_{0} \zeta \cdot \nabla (\zeta \cdot \nabla \rho_{0}) -
\]

\[
- (\zeta \cdot \nabla \rho_{0})^{2} \right] \, dx + \int \frac{1}{\rho_{0}} \left[ \rho_{2}(\zeta \cdot \nabla \rho_{0}) - \rho_{0}(\zeta \cdot \nabla \rho_{2}) \right] (\zeta \cdot \nabla P_{0}) \, dx +
\]

\[
+ 2 \int \rho_{2} [\zeta \cdot \nabla \delta \phi_{0}] \, dx,
\]

where \(\gamma_{0}(x)\) and \(\gamma_{2}(x)\) are the unperturbed and perturbed parts of the general adiabatic exponent; \(P_{0}\) and \(P_{2}\) are, respectively, the unperturbed and perturbed pressure functions; and \(\delta \phi_{0}\) denotes the change in gravitational potential of the configuration due to the presence of rotation. Furthermore, \(\zeta\) denotes the trial function in the variational formulation.

In order to make use of Equation (1) for calculating the contribution of a small uniform rotation to the frequency of various modes of pulsation, we need an equilibrium model. We consider here a perturbed partially degenerate uniformly rotating standard model for which we define the following forms for various functions (cf. Paper II)

\[
P = P_{0} + \nu P_{2} ; \quad \rho = \rho_{0} + \nu \rho_{2} ,
\]