Abstract. The evolutionary behaviour of rotating solar models with different initial angular-momentum distributions has been investigated through the pre-Main-Sequence and Main-Sequence phases. The angular momentum was removed from the convective envelope of the solar models according to the Kawaler's model of magnetic stellar wind (Kawaler, 1988). The models show that (i) the surface rotational velocities of the solar mass stars are independent of initial angular momentum for ages greater than $10^8$ years and (ii) it is not possible to explain the neutrino problem and the sufficient depletion of lithium in the Sun.

1. Introduction

It has been shown that the surface rotational velocities of stars are closely related to their masses (Kraft, 1970; Fukuda, 1982). The mean rotational velocities of the Main-Sequence stars decline steeply with decreasing stellar mass near $1.5\, M_\odot$ (spectral type F0) from $150\, \text{km s}^{-1}$ to less than $10\, \text{km s}^{-1}$ indicating a loss of angular momentum for the late-type stars. Therefore, the surface rotational velocities of stars whose masses are smaller than $1.5\, M_\odot$ do not represent their initial angular momenta.

Observations of solar type stars in the young (age $\sim 5 \times 10^7$ yr) $\alpha$ Persei cluster (Stauffer et al., 1989) show that they spin up during the contraction from the T Tauri phase (age $\sim 10^6$ yr) (Bouvier et al., 1986; Hartmann and Stauffer, 1989) to the ZAMS. A number of G stars in $\alpha$ Per cluster have the observed rotational velocities greater than $40\, \text{km s}^{-1}$. Comparison of measured rotational velocities of G stars in $\alpha$ Per, Pleiades (age $\sim 7 \times 10^7$ yr) and Hyades (age $\sim 7 \times 10^8$ yr) clusters (Stauffer and Hartmann, 1986) indicates that G stars spin down rapidly near the MS. In the Pleiades cluster, G stars have the measured rotational velocities less than $20\, \text{km s}^{-1}$ while Hyades cluster contains G stars with rotational velocities less than $10\, \text{km s}^{-1}$. Furthermore, the data indicate that G stars spin down more rapidly than K stars.

Endal and Sofia (1981), Pinsonneault et al. (1989), and MacGregor and Brenner (1990) obtained the rotational evolutionary models of the Sun up to the present time by considering the redistribution of angular momentum within the interior of the Sun. The angular momentum was removed from the convective envelope of the Sun according to the Kawaler's model of magnetic stellar wind (Kawaler, 1988) in the study of Pinsonneault et al. (1989). In the other two studies, the Belcher and MacGregor (1976) model of solar wind was used in removing the angular momentum from the outer convective layers.

In the present study, the evolutionary behaviours of rotating solar models in the presence of angular-momentum loss by magnetic stellar wind were investigated through both PMS and MS phases. Kawaler's wind model was applied to the evolutionary
models to see if such wind model could account for the loss of angular momentum in the Sun. Although it is not realistic, the limiting case of no angular-momentum coupling between the core and the envelope is assumed. Therefore, convective envelope rotates slowly while the radiative core is in rapid rotation up to \( 0.8 R_\odot \) at the solar age. This contradicts the observational evidence that the solar rotation is almost uniform and slow, down to much deeper layers (Hill and Stebbins, 1975; Brown et al., 1989). The redistribution of angular momentum within the interior of the Sun will be considered in a future study. The procedure followed in obtaining models is in the next section. The last section presents the results.

2. Models and Method

One solar mass star models were evolved up to the present age of the Sun, starting from the threshold of stability at which the gravitational potential energy balances the internal energy of the contracting gas cloud, assuming that the rotation is negligible at this point. Two different initial angular-momentum distribution were estimated in order to see the consequences of the variation of this uncertain parameter. Three evolutionary tracks for the Sun with initial angular-momentum distributions \( J_0 = 1.4 \times 10^{50} \text{ g cm}^2 \text{s}^{-1} \) (Sequence B and Sequence C) and \( J_0 = 2.3 \times 10^{50} \text{ g cm}^2 \text{s}^{-1} \) (Sequence A) were obtained by allowing the angular-momentum loss from the surface. As the models were evolved, angular momentum by magnetic stellar winds was removed from the convective envelope following the equation given by Kawaler (1988) as

\[
\frac{dJ}{dt} = -K_w W^{1 + 4\eta/3} \left( \frac{R}{R_\odot} \right)^2 \left( \frac{M}{M_\odot} \right)^{-\eta/3} \left( \frac{\dot{M}}{10^{-14}} \right)^{1 - 2\eta/3};
\]

\( K_w \) being a constant that combines scale factors for the wind velocity and magnetic field strength. By changing this constant, the above equation was calibrated to obtain solar luminosity, radius and solar surface rotational velocity of \( 2 \times 10^3 \text{ cm s}^{-1} \) at the solar age; \( n \) determines the magnetic field geometry and \( a \) gives information about the relation between the surface magnetic field strength \( B \) and angular velocity \( W(B a W^a) \) as described by Kawaler (1988), \( \dot{M} \) is the mass loss rate.

The hydrogen and heavy element abundance, by mass, were taken as \( X = 0.760, Z = 0.019 \) for Sequence A and Sequence C evolutions. The correct value of present-day solar luminosity and radius were not obtained for this composition, therefore, a lower value of the initial hydrogen content was considered: \( X = 0.750, Z = 0.019 \). The evolutionary calculations were repeated with initial angular momentum \( J_0 = 1.4 \times 10^{50} \text{ g cm}^2 \text{s}^{-1} \) (Sequence B evolution) to obtain the right values for the present Sun. In this study, \( n \) and \( a \) were chosen as 1.5 and 1. The present-day solar values were obtained for Sequence B models with \( K_w = 1.1 \times 10^{46} \text{ g cm}^2 \text{s} \).

Rotation was treated by taking convenient averages of the rotational effects on the spherical shells; that is the physical variables of the radius \( r \) were taken to be averaged variables over the spherical shell of mean radius \( r \). The basic structure equations which has been modified to take into account of the changes due to rotation, the method of