SOLUTION OF THE EQUATION OF TRANSFER FOR
COHERENT SCATTERING IN AN EXPONENTIAL
ATMOSPHERE BY BUSBRIDGE'S METHOD

T. K. DEB
Department of Telecommunications, Siliguri, West Bengal, India

and

S. KARANJAI
Department of Mathematics, North Bengal University, West Bengal, India

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Abstract. A solution of the transfer equation for coherent scattering in stellar atmosphere with Planck's
function as a nonlinear function of optical depth, viz.

$$B_\nu(T) = b_0 + b_1 e^{-\beta\tau}$$

is obtained by the method developed by Busbridge (1953).

1. Introduction

Chandrasekhar (1960) applied the method of discrete ordinates to solve the transfer
equation for coherent scattering in stellar atmosphere with Planck's function as a linear
function of optical depth, viz.,

$$B_\nu(T) = b_0 + b_1 \tau .$$

The equation of transfer for coherent scattering has also been solved by Eddington's
method (when $\eta_\nu$, the ratio of line to the continuum absorption coefficient is constant)
and by Strömgren's method (when $\eta_\nu$ has small but arbitrary variation with optical
depth; see Woolley and Stibbs, 1953). Busbridge (1953) solved the same problem by
a new method using Chandrasekhar's ideas. Dasgupta (1977b) applied the method of
Laplace transform and Wiener–Hopf technique to find an exact solution of the transfer
equation for coherent scattering in the stellar atmosphere with Planck's function as a
sum of elementary functions, viz.,

$$B_\nu(T) = b_0 + b_1 \tau + \sum_{r=2}^{n} b_r E_r(\tau) ,$$

using a new representation of the $H$-function obtained by Dasgupta (1977a). Recently,
Karanjai and Deb (1991a, b) solved the equation of transfer for coherent isotropic
scattering in an exponential atmosphere by Eddington's method and the method of
Laplace transform and Wiener–Hopf technique. In this paper, we have obtained a
solution of the equation of transfer for coherent scattering in an exponential atmosphere,
i.e.,

\[ B_v(T) = b_0 + b_1 e^{-\beta \tau}; \]

where \( b_0, b_1, \) and \( \beta \) are three positive constants, by the method used by Busbridge (1953).

### 2. Equation of Transfer

With the usual notation of transfer for the Milne–Eddington model can be written (Busbridge, 1953; Chandrasekhar, 1960) as

\[
\mu \frac{dI_v}{\rho \, dz} = (k_v + \sigma_v) I_v - \frac{1}{2} \sigma_v \int_{-1}^{+1} I_v \, d\mu' - k_v B_v(T),
\]

where \( z \) is the depth below the surface; \( k_v \), the continuous absorption coefficient; and \( \sigma_v \) is the line-scattering coefficient. We assume that \( k_v \) and \( \sigma_v \) are independent of depth and we write

\[
t = \int_0^z \rho(k_v + \sigma_v) \, dz,
\]

\[
\tau = \int_0^z \rho k_v \, dz,
\]

\[
\eta_v = \frac{\sigma_v}{k_v}, \quad \lambda_v = \frac{1}{1 + \eta_v} = \frac{k_v}{k_v + \sigma_v}.
\]

Then

\[ \tau = \lambda_v t \]

and

\[ B_v(T) = b_0 + b_1 e^{-\beta \tau} = b_0 + b_1 e^{-\beta \lambda_v t}, \]

where \( B_v(T) \) is the Planck's function.

Substituting into Equation (1), we get

\[
\mu \frac{dI_v}{d\tau_v} = I_v(t, \mu) - \frac{1}{2}(1 - \lambda_v) \int_{-1}^{+1} I_v(t, \mu') \, d\mu' - \lambda_v (b_0 + b_1 e^{-\beta \lambda_v t}).
\]

Equation (5) has to be solved subject to the boundary conditions

\[ I_v(0, -\mu') = 0, \quad (0 < \mu' < 1) \]