COSMOLOGICAL VISCOUS FLUID UNIVERSE IN SLOW
ROTATION

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Abstract. The problem of slowly rotating cosmological viscous fluid universe in a homogeneous and isotropic models has been investigated by considering the perturbation in the metric rotation function to the first order of smallness associated with certain physical restrictions imposed on the metric rotation function and matter angular velocity. Some more general solutions for the metric rotation function have been obtained and physical interpretation of the solutions have been investigated.

1. Introduction

Since there has been some observational evidence as to the rotation of the Universe in some sense, several authors have studied the problem of slowly rotating viscous fluid universe in cosmological models. Fennelly (1975) has shown that since the model is considered to be spatially-homogeneous, the whole Universe would have a rotational motion even when there exists no centre of rotation. Gödel (1949, 1952) deduced for the first time an exact rotating cosmological solution of the Einstein field equations for the rotating matter. Cohen and Brill (1968), Adam et al. (1973, 1974), Whitman and Pizzo (1979), Bayin (1981) studied the properties of solutions of slowly rotating perfect fluid in general relativity. It also has been observed that all neutron stars are found to satisfy the condition of slow rotation thereby showing that slowly rotating perfect fluid solutions can be treated as the mathematical models for neutron stars. Since the Einstein's field equation are found to be highly nonlinear partial differential equations for a rotating perfect fluid the authors have to study the slowly rotating solutions by taking approximations to the first order in the metric rotation function \( \Omega(r, t) \). Tarachand and Singh (1987) have studied the problem of slowly rotating cosmological fluid spheres. Singh and Bhamra (1986) have derived analytical solutions for rotational perturbations of the Robertson–Walker metric incorporating the possibility of the existence of rotating viscous fluid under two physical restrictions. Tarachand and Ibotombi (1988) have further investigated the problem of slowly rotating cosmological viscous fluid universe by imposing different physical restrictions and exact solutions for \( \Omega(r, t) \) have been obtained for cosmological models.

In the present paper the very problem of slowly rotating cosmological viscous fluid universe has been investigated and some other general solutions for \( \Omega(r, t) \) have been obtained under different physical restrictions by considering rotational perturbations of the Robertson–Walker metric. Analytical solutions have been derived by considering imperfect fluid and non-vanishing bulk viscosity coefficient. For the mathematical formulation of the problem we make the following assumptions:
(i) The cosmological models are taken to be homogeneous and isotropic.
(ii) There is no interaction other than the gravitation and that due to the presence of viscous fluid.
(iii) Space-time of equilibrium of configuration is axially symmetric.
(iv) The observer is supposed to be in the substratum at the centre of distribution of small rotation.
(v) The perturbations of slow and uniform rotation function of the cosmological metric are taken to allow the justification in neglecting all the terms containing second and higher orders of rotational velocity $\Omega(r, t)$.

We have obtained in this paper the solutions for $\Omega(r, t)$ without imposing the physical restrictions and also by imposing the physical restrictions that (i) $\omega = a\Omega$ and (ii) $\Omega - \omega = A(r)$ where $a$ is an arbitrary constant and $\omega$ is the matter angular rotation function of the radial distance $r$ and the coordinate time $t$.

In Section 2 we have presented the mathematical formulation of the problem and in Section 3 the surviving Einstein’s field equations are derived by taking approximation to the first order in $\Omega(r, t)$ and general solutions for $\Omega(r, t)$ corresponding to several physical assumptions are obtained and the physical interpretations of the solutions are presented. In Section 4 a general discussion of the results is presented. In Section 5 we have given a general conclusion of the results obtained in all the three cases.

2. Mathematical Formulation

On the analogy of Lausberg’s (1969) argument the cosmological metric to be perturbed is taken as
\[
\text{ds}^2 = \text{dt}^2 - R^2(t) [(1 - Kr^2)^{-1} \text{dr}^2 + r^2 (d\theta^2 + \sin^2 \theta) (d\phi - \Omega \text{dt})^2] , \quad (1)
\]
where $\Omega(r, t)$ represents the angular velocity of the inertial frame along the axis of rotation.

On account of the slow rotation we have $C^2 \geq \omega R^2(t) > R^2(t)\Omega^2$ and, therefore, proper approximation can be taken up to the first order of an angular velocity $\Omega(r, t)$. Hence, the perturbed metric can be taken in the form as
\[
\text{ds}^2 = \text{dt}^2 - R^2(t) [(1 - Kr^2)^{-1} \text{dr}^2 + r^2 (d\theta^2 + \sin^2 \theta) d\phi^2] + 2\Omega(r, t)R^2r^2 \sin^2 \theta - d\phi \text{dt} . \quad (2)
\]

Einstein’s general field equations are given by
\[
G_{ij} = R_{ij} - (\frac{1}{2})R g_{ij} + \Lambda g_{ij} = -8\pi T_{ij} , \quad (3)
\]
where the energy-momentum tensor $T_{ij}$ of the viscous fluid is given by
\[
T_{ij} = (\rho + p - \zeta \theta)U_iU_j - (P - \zeta \theta)g_{ij} - 2\eta \sigma_{ij} , \quad (4)
\]
in which $p$ is the isotropic pressure; $\rho$, the matter density; $U^i$, the four-velocity vector; $\eta$ and $\zeta$ are the coefficients of shear and bulk viscosity, respectively. Here $\sigma_{ij}$ is the shear...