SECULAR VARIATIONS IN THE RESTRICTED PROBLEM:
A NUMERICAL TECHNIQUE FOR STUDYING STABILITY
OF EQUILIBRIUM SOLUTIONS

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(Received 11 April, 1989)

Abstract. In this paper we develop and implement an explicit numerical technique for studying stability of
equilibrium solutions concerning the secular variations in the restricted problem. In our implementation,
we use fourth-order expansions for the secular terms, but the method itself is independent of the particular
order used as upper limit in the required expansions.

1. Introduction

In a recent paper (Geroyannis and Flogaitis, 1989; hereafter referred to as Paper I) we
developed an explicit numerical method for studying the secular variations in the
restricted problem. This method consists in a systematic scanning of the $\alpha - H$ plane,
where $\alpha$ denotes the semi-major axis and $H$ stands for the Delaunay variable. By such
an explicit scanning, the equilibrium solutions, assigned to a particular point $(\alpha, H)$ of
the $\alpha - H$ plane, are localized as roots of a system of two highly nonlinear algebraic
equations (Paper I, Equations (2.62a, b)). These equations result from the requirement
that the partial derivatives of the modified Hamiltonian $F^*(\alpha^{1/2}, H; p, q)$ in the two
fundamental variables $p$ and $q$ (Paper I, Equations (2.19), (2.14a), and (2.14b),
respectively) must be equal to zero in order for the roots $\bar{p}$ and $\bar{q}$ to represent equilibrium
solutions. In addition, the roots $\bar{p}$ and $\bar{q}$, evaluated by a rootfinding algorithm, must
satisfy four necessary conditions (Paper I, Section 3, Equations (3.7)-(3.19)). For a
pure numerical-analysis viewpoint, root estimates $\bar{p}$ and $\bar{q}$ satisfying these conditions are
'Acceptable Equilibrium Solutions' for the equations of motion (Paper I, Equa-
tions (2.61a, b)), abbreviated AES, and denoted by $\bar{p}$ and $\bar{q}$. A general display of such
solutions is given in Figure 1.

However, the problem of distinguishing 'stable' and 'unstable' AES remains open.
The stability analysis required for the higher-order terms of our equations seems to
present great difficulties when viewed according to Liapounov's methodology
(Liapounov, 1907). Therefore, we develop and implement in the present paper an
'Explicit Numerical Stability-Analysis Technique', abbreviated ENSAT, which fits very
well the framework of the numerical strategy of Paper I. Moreover, ENSAT seems to
be a general technique, applicable to other similar stability problems.

For clarity and convenience, we shall use hereafter the definitions and symbols of
Paper I.
2. The Numerical Technique

To develop ENSAT, we need first to focus on the procedure by which we drive the rootfinding algorithm. In detail, for a given point \((\alpha, H)\) of the \(\alpha - H\) plane we feed the rootfinder with guess points \((p_j, q_{jk})\) taken from a two-dimensional optimum guess grid 

\[
G_{pq} = \{p_j\} \otimes \{q_{jk}\}
\]

of the \(I_p - I_{aj}\) plane, where

\[
I_p = [ - \tau^{1/2}, + \tau^{1/2} ],
\]

(2.1)