SOLUTION OF A GENERALIZED EQUATION OF TRANSFER IN A TWO-REGION SLAB

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Abstract. The general equation of transfer in a two-region slab of unequal thickness with general boundary conditions has been solved by an analytical method developed by Menning et al. (1980). The scattering is regarded as isotropic and the source function is taken as a general one to accommodate different types of problems.

1. Introduction

Menning et al. (1980) developed a power series expansion method (PS method) for solving the multigroup transport equation in plane-parallel geometry. In this method the radiation intensity was separated into forward and backward components which were then expanded in powers of space-variables while the angular variable was treated exactly. The consideration of generalised boundary conditions accommodated various types of problems such as radiative heat transfer, neutron transport, etc. They derived analytical expressions for the angular distribution and the angle-integrated intensities by solving a matrix problem in which the matrix elements were integrals over rational functions of angular variables. Thus the method is useful where the intensity function is strongly dependent on angular variables. Pomraning and Clark (1964) applied a Legendre polynomial expansion method (L_n method) to some reactor eigenvalue problems which is, to some extent, similar to the PS method.

In this paper the general equation of transfer for isotropic scattering in a two-region slab of unequal thickness with general boundary conditions has been solved by using the PS method.

2. Formulation of the Problem

The particular problem considered here consists of a two-layer slab medium (1 and 2), such that \( l_1 \leq t \leq l_2, l_2 \leq t \leq l_3 \) in physical space variables \( t \). Let \( \beta_1 \) and \( \beta_2 \) are extinction coefficients (Shoumen and Özisik, 1981) for regions 1 and 2, respectively. It is now convenient to define optical variable \( X \) as,

\[
X = \beta_1 t \quad \text{for } l_1 \leq t \leq l_2 = (\beta_1 - \beta_2)\frac{1}{2}a + \beta_2 t \quad \text{for } l_2 \leq t \leq l_3
\]

Hence,

\[-a = l_1 \beta_1 \leq X \leq l_2 \beta_2 = \frac{1}{2}a \quad \text{for region 1},
\]

\[
\frac{1}{2}a = l_2 \beta_2 \leq X \leq l_3 \beta_3 = a \quad \text{for region 2}.
\]
The appropriate transfer equation for generalized boundary condition

\[ \mu \frac{\partial}{\partial x} I_\alpha(X, \mu) + I_\alpha(X, \mu) = S_\alpha(X) + \frac{w_\alpha}{2} \int_{-1}^{+1} I_\alpha(X, \mu) \, d\mu \]  

for \( \alpha = 1, 2 \), where \( S_\alpha(X) \) represents the source term. For different cases this source term can take different expressions. For radiative heat transfer problems source function may take the form (Shoumen and Özisik, 1981)

\[ S_\alpha(X) = (1 - w_\alpha) \frac{\sigma T_\alpha^d(X)}{\pi}, \]

where \( T(X) \) is the temperature of the medium, \( \mu \) represents direction cosine, \( w_\alpha \) is the single-scattering albedo for the medium.

The generalized boundary conditions are given by

\[ I_1(-a, \mu) = f_1 + \rho_{1d} I_1(-a, -\mu) + 2\rho_{1s} \int_{0}^{1} \mu' I_1(-a, -\mu') \, d\mu', \]  

\[ I_1(\frac{1}{2}a - \mu) = f_2 + \gamma_1 I_2(\frac{1}{2}a, -\mu) + \rho_{2s} I_1(\frac{1}{2}a, \mu) + \]  

\[ + 2\rho_{2d} \int_{0}^{1} \mu' I_1(\frac{1}{2}a, \mu') \, d\mu', \]  

\[ I_2(\frac{1}{2}a, \mu) = f_3 + \gamma_1 I_1(\frac{1}{2}a, \mu) + \rho_{3s} I_2(\frac{1}{2}a, -\mu) + \]  

\[ + 2\rho_{3d} \int_{0}^{1} \mu' I_2(\frac{1}{2}a, -\mu') \, d\mu', \]  

\[ I_2(a, -\mu) = f_4 + \rho_{4s} I_2(a, \mu) + 2\rho_{4d} \int_{0}^{1} \mu' I_2(a, \mu') \, d\mu', \quad \mu, \mu > 0. \]

In these equations the \( f \)'s represents diffusely-emitting sources, \( \rho_d \) and \( \rho_s \) refer, respectively, to diffuse and specular reflectivity and \( \gamma \) to interface transmissivity. The following figure shows the location of different quantities stated above.