HYDROMAGNETIC STABILITY OF A GAS-LIQUID INTERFACE
OF COAXIAL CYLINDERS

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Abstract. The magnetohydrodynamic stability of a finite hollow jet acting it up on the capilarity, inertia and electromagnetic forces has been developed. A general eigenvalue relation is derived and studied analytically and the results are confirmed numerically and interpreted physically to all possible modes of perturbations. The model is capillary unstable to small axisymmetric disturbance and stable for all other disturbances. The model is magnetodynamic stable to all possible modes of perturbations for all (short and long) wavelengths. The magnetic field exerts a strong stabilizing influence not only to axisymmetric disturbance which causes only the bending of the magnetic lines of force but also to the non-axisymmetric disturbances that also leads to twisting of lines. It is found that the capillary instability is quickly decreasing with increasing the magnetic field intensities and, moreover, above a certain value of the basic magnetic field the capillary instability is completely suppressed and stability sets in.

Numerous reported works are recovered as limiting cases from the present general work.

1. Introduction

The (in)-stability of full liquid jets has been treated comprehensively experimentally, analytically, and numerically for a long time. This was due to its crucial applications in miscellaneous domains. Research in this field intensified when it became apparent that the physical properties of liquid jets play an essential part in a rapidly growing number of applications ranging from the designing of sprays to the inkjet printers.

Plateau (1873) was the first to obtain the capillary critical wavelength experimentally and theoretically utilizing a naïve approach. The decisive breakthrough came with Rayleigh (1945) who devised and laid inexorable foundations for the theoretical treatment of such and related problems and, moreover, he developed the most important concept of the maximum mode of instability. Chandrasekhar (1981) made numerous and many extensions for different models under the action of several and different forces. The effect of nonlinearities on the capillary instability of a liquid jet was investigated by Wang (1968), Nayfeh (1970), and Hassan (1971) and a complete analysis was finally elaborated by Kakutani et al. (1974).

The instability of an annular liquid jet (i.e., a liquid jet having a gas jet as a mantle) is also interesting to investigate (Kendall, 1986; Lee and Wang, 1986). Such stability due to the capillarity force has been considered experimentally by Kendall (1986) and theoretically by Lee and Wang (1986). It has an important correlation with the potential utility of a method for the mass production rigid shell of high quality (Kendall et al., 1982).

Indeed, Kendall (1986) explained clearly and speculatively, in a large context, about the practical applications of that model in the astrophysics domain. Moreover, he did

attract and drew the attention for the theoretical studies of the stability of such model. Recently, Radwan has investigated the stability of a (viscous (Radwan and Elazab, 1987)) and streaming (Radwan, 1989) hollow jet (i.e., a gas jet submerged in an infinite liquid) under the influence of the capability force.

The endeavours of the present work investigating the magnetohydrodynamic stability of a gas jet ambient with a liquid whether it (liquid) is finite or infinite acting up on the capilarity, inertia, and electromagnetic forces.

2. Formulation of the Problem

Consider a gas jet of radius $R_0$ surrounded by a liquid of uniform density $\rho$ of radius $qR_0$ where $1 < q < \infty$, in equilibrium state. The gas jet is assumed to be pervaded by the magnetic field $(0, 0, \alpha H_0)$ and the liquid by $(0, 0, H_0)$ where $H_0$ is the basic magnetic field and $\alpha$ the parameter satisfies some restrictions, see Equation (14). The fluids are assumed to be incompressible, inviscid, and perfectly conducting. They are acting upon the capilarity, inertia, and electromagnetic forces such that, following Kendall (1986), the liquid inertia force is paramount over that of the gas jet. We shall use the cylindrical coordinates $(r, \varphi, z)$ system with the $z$-axis is coinciding with the axis of the coaxial cylinders. Note that as an approximation to Kendall’s (1986) work model, we have taken that the liquid surrounding the gas jet is termined by a solid cylindrical surface with radial distance $qR_0$ with $1 < q < \infty$.

The basic equations are the combination of the hydrodynamic equations and those of Maxwell concerning an electromagnetic system. For the problem under consideration in the liquid they are:

\[ \rho \left( \frac{\partial}{\partial t} + (u \cdot \nabla) \right) u = -\nabla p + j \times B, \quad (1) \]

\[ \nabla \cdot u = 0, \quad (2) \]

\[ \frac{\partial B}{\partial t} = - (\nabla \times E), \quad (3) \]

\[ \nabla \cdot B = 0, \quad (4) \]

\[ E + (u \times B) = 0, \quad (5) \]

\[ j = \frac{1}{\mu} (\nabla \times B), \quad (6) \]

in the gas

\[ \nabla \cdot H^g = 0, \quad (7) \]

\[ \nabla \times H^g = 0; \quad (8) \]