SELF-SIMILAR FLOWS BEHIND M.H.D. SHOCK WAVES

(Letter to the Editor)

P. RAM
Department of Mathematics, S.G.R. Postgraduate College Dobhi, Jaunpur, India

and

D. D. DUBE
K.S. Saket Postgraduate College Ayodhya, Faizabad, India

(Received 5 May, 1989)

Abstract. In this paper a self-similar motion in a medium of infinite electrical conductivity has been investigated in spherical symmetry under the influence of an idealized magnetic field and a comparison has been made of the state of flow variables with and without magnetic field.

1. Introduction

Assuming the total energy of the flow behind the shock surface has been studied by Rogers (1958), Ranga Rao and Purohit (1972), and Singh (1980). The fact that the material within a star is a plasma of infinite electrical conductivity and that it exists within a strong magnetic field have led us to study the interaction of magnetic field with the other flow variables. Summers (1975) has discussed a model in which he has considered an idealized field such that the lines of force lie on a hemisphere whose centre is the point of explosion and directed tangentially to the advancing shock front.

We have investigated the state of flow variables behind a strong M.H.D. shock in a heterogeneous medium of infinite electrical conductivity and to compare the findings with the state of flow variables when the magnetic field is ignored. We have used Summer's (1975) model in writing the equations depicting the interaction of idealized magnetic field with other gasdynamic variables.

The non-uniform ambient medium into which shock wave advances is characterized as

\[ \rho_0 = \rho_e R^{-\alpha} \quad (\alpha > 0), \]

where \( \rho_0 \) is a variable density of ambient medium at rest; \( \rho_e \), a constant; and \( R \), the radius of shock surface depending on time alone, obeying the power law

\[ R = \eta e^{\beta t}, \]

where \( \beta \) is an arbitrary constant. The similarity variable required, to reduce the equations into ordinary differential equations (Rogers, 1957) has been taken as

\[ \eta = r e^{-\beta t}, \]

whereas at the shock surface $\eta$ assumes the value $\eta = \eta_c$. The velocity of the shock surface $V$ is, therefore, given by

$$V = \dot{R} = \beta R.$$  \hfill (4)

The total energy $E$ of the shock wave in between inner expanding vacuous boundary and shock boundary has been assumed to be increased according to the power law

$$E = E_c R^\delta \quad (\delta > 0),$$  \hfill (5)

where $E_c$ is a constant.

The gravitational forces, viscosity, heat conduction, and radiation have been neglected.

2. Basic Equations and Boundary Conditions

The basic equations in Eulerian coordinate system under the assumption of local radiality governing M.H.D. flows (cf. Summers, 1975) are

$$\frac{D\rho}{Dt} + \rho \frac{\partial u}{\partial r} + \frac{2\rho u}{r} = 0,$$  \hfill (6)

$$\frac{Du}{Dt} + \frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{H}{\rho} \frac{\partial H}{\partial r} + \frac{H^2}{\rho r} = 0,$$  \hfill (7)

$$\frac{DH}{Dt} + H \frac{\partial u}{\partial r} + \frac{Hu}{r} = 0,$$  \hfill (8)

$$\frac{D}{Dt} (p \rho^{-\gamma}) = 0,$$  \hfill (9)

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial r},$$

and $\gamma$ is the ratio of specific heats and the gas is approximately ideal, $u$, $\rho$, $p$, $H$, $r$, and $t$ are, respectively, flow velocity, density, pressure, azimuthal magnetic field, position of the fluid in radial length and time.

If we apply strong shock approximations at the shock boundary, the jump conditions for the flow variables and magnetic field are given by

$$u_1 = \frac{2}{\gamma + 1} V,$$

$$p_1 = \frac{2}{\gamma + 1} \rho_0 V^2,$$

$$\rho_1 = \frac{\gamma + 1}{\gamma - 1} \rho_0$$

and

$$H_1 = \frac{\gamma + 1}{\gamma - 1} H_0,$$  \hfill (10)